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Completing the framework of AdS/QCD: h_1/b_1 mesons and excited ω/ρ's

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Abstract: We extend the "hard wall" gravity dual of QCD by including tensor fields b_{MN} that correspond to the QCD quark bilinear operators $\bar{q}\sigma^{\mu\nu}q$. These fields give rise to a spectrum of states which include the h_1 and b_1 mesons, as well as a tower of excited ω/ρ meson states. We also identify the lowest-dimension term which leads to mixing between the new ρ states and the usual tower of ρ mesons when chiral symmetry is broken.

Keywords: AdS-CFT Correspondence, AdS/QCD

Contents

1 Introduction and motivation

The last few years have seen a renewed focus on string dual descriptions of QCD following the discovery of the AdS/CFT correspondence [1–3] and subsequent attempts to construct holographic string/gravity duals of QCD in both top-down [4, 5] and bottom-up approaches [6–10]. These models have been remarkably successful in reproducing some of the low-energy features of QCD, but also suffer a number of flaws discussed at length in the literature [12–15].

For concreteness, let us consider the "hard wall" model of [7]. This model contains five-dimensional (5d) fields $A^a_{L,\mu}$, $A^a_{R,\mu}$ and $X^{\alpha\beta}$ dual to the operators $\bar{q}_L\gamma_\mu t^a q_L$, $\bar{q}_R\gamma_\mu t^a q_R$ and $\bar{q}^\alpha_R q^\beta_L$ respectively. These fields live in a 5d Anti de Sitter (AdS) space with a hard cutoff on the radial coordinate at $z = z_0$ with $1/z_0 = 346$ MeV. The model gives rise to a spectrum of 4d fields whose overall scale is determined by the infrared (IR) cutoff z_0. The lightest modes of definite parity are the π, ρ and a_1 mesons; the model generates an excellent fit to the masses, decay constants and coupling constants of these mesons in terms of three parameters: z_0, the quark mass $m_q = 2.3$ MeV, and the scale of chiral

symmetry breaking $\sigma = (308\ \text{MeV})^3$ (using the Model B values of [7]). Other "bottom-up" models [8–10] have a very similar structure. The "top-down" model of [4] is based on a D-brane construction in string theory, and whose field content is uniquely as a result. It is roughly as successful at predicting the properties of the low-lying mesons as the bottom-up approaches.

Despite its successes, the hard wall model suffers from a variety of issues, some of which we address in this paper. The most obvious problem has to do with the structure of the excited states in the meson spectrum. The first excited ρ or ω meson state is predicted to lie at $m(\rho') = x_{0,2}/z_0 = 1910$ MeV where $x_{0,2}$ is the second zero of the Bessel function $J_0(x)$, while in reality the mass of the observed excited state is at roughly 1450 MeV. In addition, the mass squared of higher excited states scales with the excitation number as n^2 rather than as n as expected from Regge theory and semi-classical arguments [12, 16].

One solution to the problem of the excited meson spectrum, analyzed in [16], is to alter the z-dependence of the metric and of the dilaton profile. In [16] a simple modification was suggested, which leads to a spectrum of excited vector mesons of the form $m_n^2 = \Sigma(n+1)$ where Σ is a constant related to the QCD string tension. For an appropriate choice of Σ this fits the low-lying spectrum reasonably well and leads to the expected asymptotic behavior at large n.

However, we will see that this cannot be the full story. The hard wall model does not even fully describe the light meson spectrum. It gives reasonable predictions for the lowest-lying isotriplet vector and axial-vector mesons, the ρ and a_1 (and also the isosinglet ω and f_1 if the gauge group is generalized from $SU(2)_L \times SU(2)_R$ to $U(2)_L \times U(2)_R$) but does not include fields which give rise to the $J^{PC} = 1^{+-}$ h_1 and b_1 mesons whose masses, at ~ 1200 MeV, lie just below those of the a_1 and f_1. Furthermore, as discussed in [11–13], from QCD one actually expects two types of ρ mesons. The first type couples dominantly to the usual vector current $\bar{q}\gamma^\mu t^a q$, the second to the tensor operator $\bar{q}\sigma^{\mu\nu} t^a q$. In the limit of unbroken chiral symmetry these two operators lie in different representations of the chiral symmetry group, leading to two distinct towers of states with distinct (chiral) quantum numbers. When chiral symmetry is broken, the operators mix and the physical ρ mesons will be linear combinations of the two ρ varieties. One cannot expect to get a correct spectrum of excited ρ and ω mesons if this structure is ignored.

A possible solution for including the h_1 and b_1 mesons was suggested in [17]; one should add 5d fields dual to the interpolating operators for these mesons. These operators are of the form $\bar{q}\sigma^{\mu\nu} t^a q$; incorporating their dual fields is a very natural generalization of the hard wall and other dual models of QCD. Once one includes fields dual to some of the canonical dimension three operators in QCD as was done in [7] it is natural to include fields dual to *all* the canonical dimension three operators in QCD. Although the solution is conceptually clear, we will need to overcome a number of technical issues in order to implement this idea.

In what follows we will argue that there is a natural extension of the hard wall model which includes complex, antisymmetric tensor fields b_{MN} transforming in the bifundamen-

tal of the $U(N_f)_L \times U(N_f)_R$ gauge symmetry, that are dual to the operators $\bar{q}\sigma^{\mu\nu}t^a q$[1]. We will show that these fields have a natural first order action, familiar from the description of charged tensor fields in $AdS_5 \times S^5$ [18, 19], and that their expansion in terms of 4d fields leads to a tower of mesons with the quantum numbers of the h_1 and b_1 as well as an additional tower of mesons with the same quantum numbers as the ρ and ω mesons. We will also identify the lowest dimension operator which induces mixing between the two distinct towers of ρ/ω mesons in the presence of chiral symmetry breaking, and which is also responsible for decay processes like $h_1 \to \rho + \pi$ and $b_1 \to \omega + \pi$. Our treatment follows the standard formalism of string/gauge duality. In this we differ from [20] where it is claimed that modifications to the formalism involving the asymptotic behavior of fields is required in order to incorporate tensor mesons in the presence of chiral symmetry breaking. Our description of tensor mesons also seems to be somewhat at odds with the description given in [21]. It is not clear to us whether the formalism used in [21] included the possibility of fields with a first order Lagrangian. [16] does not address this additional tensor operator, and in fact we should note that despite its encouraging fit with the excited ρ spectrum, [16] does not address the h_1 and b_1 states. Once we include fields dual to these excitations we necessarily generate additional vector meson states, presumably ruining the agreement between the excited ρ's of [16] and experiment.

In addition to these specific issues, which are directly addressed in the body of the paper, our analysis will also touch on some important problems of principle that arise in the construction of QCD duals. These involve the nature the $1/N_c$ expansion, the validity of a local 5d field theory description and the field theory interpretation of the asymptotically AdS spacetime.

Let us consider the last point first. It is often said that the conformal invariance of Anti de Sitter space matches the conformal invariance that QCD enjoys in the ultraviolet (UV) because of its asymptotic freedom. This is not entirely correct. In the best-understood example, the AdS/CFT correspondence, the conformal isometry of AdS space is in accordance with the conformal invariance of $N = 4$ SYM at large 't Hooft coupling. It is important to note that in this limit $N = 4$ SYM is *not* a free field theory, but is rather a nontrivial CFT with dimensions and correlators which differ from those of free field theory. In the analysis of [7] the 5d gauge coupling is determined by matching onto the UV behavior of the vector current two point function in QCD, which can be computed reliably from a one-loop diagram because of asymptotic freedom. This is a very special case, however, because the anomalous dimension of the (conserved) vector current vanishes. In contrast, the tensor operator $\bar{q}t^a\sigma_{\mu\nu}q$ is not conserved and has a nonzero anomalous dimension. We will see for example that matching its anomalous dimension as derived from the AdS/CFT correspondence to the UV behavior of QCD leads to an unphysical value of the mass of the lowest b_1/h_1 mesons. This suggests that the correct interpretation of models like [7] is that they are modeling QCD as a theory which has a "conformal window" of energy scales

[1]We work in the context of the hard wall model for simplicity. A more complete dual description of QCD will undoubtedly incorporate the tensor fields discussed here as well as modifications of the metric and dilaton profile in order to produce the correct asymptotic behavior of excited states. The dual description of the thermodynamic properties of QCD also requires such modifications. See e.g. [22] for a recent review.

in which the theory behaves as a nontrivial CFT and then has conformal invariance broken at an IR scale $1/z_0$. This is clearly not the behavior of real world QCD, but apparently for some purposes it is a reasonable approximation. With this picture in mind we can obtain the appropriate 5d mass of the tensor field (dual to the anomalous dimension of the tensor operator) required to fit the observed mass of the b_1/h_1 mesons.

We turn now to the issue of a local 5d dual description of QCD. In current models one studies a 5d action for massless fields such as the gauge fields $A_{L,R}$. In the top-down model of [4] these fields arise as the lightest open string modes. One can also include the lightest closed string modes: the graviton, dilaton and antisymmetric tensor fields, which provide a dual description of the glueball spectrum of QCD. One then either writes down the lowest dimension interactions involving these fields, or derives the lowest dimension terms using an α' expansion in string theory. In other words, one works in a "supergravity" limit of the full string dual. Such an approximation is usually justified only if there is a clear separation of scales between the massive string modes and the energy scale being used to probe the massless string modes. There is no evidence for such a separation of scales in real world QCD. For example, the string scale extracted from the Regge trajectories of meson states is $\alpha' \simeq 0.88$ GeV^2 which leads us to expect massive string modes at a scale of order $1/\sqrt{\alpha'} \simeq 1$ GeV. Yet in both the top-down and bottom-up models this scale is comparable to the scale of excited states of the massless modes. Put another way, in the current models one keeps only the lowest dimension operators constructed out of the massless modes. If there is no separation of energy scales, there is no reason to expect higher dimension operators to be suppressed, or for that matter, no reason to expect that we can use a local 5d field theory description at all.

Nevertheless, in what follows we assume that it makes sense to use a local 5d field theory description. One possible theoretical justification for this has been suggested in [16, 23]. [23] proposes that there exists a local 5d description in the $N \to \infty$ limit for small 't Hooft coupling, $\lambda = g^2 N$, even though this is the $\alpha' \to \infty$ limit rather than the $\alpha' \to 0$ limit one usually considers to obtain a local 5d action, and that, furthermore, the existence of such a local description is linked to the existence of an infinite family of conserved tensors of arbitrarily high spin. See [24] for a review.

In [16] it was argued that one can add 5d fields dual to an infinite family of twist two operators in QCD of the form $\bar{q}\gamma^{(\mu_1} D^{\mu_2} \cdots D^{\mu_n)} q$ while maintaining general coordinate and tensor gauge invariance at quadratic order in the fields. The tensor field b_{MN} that we add is also dual to a twist two operator in QCD, and we will show that again one can maintain general coordinate and gauge invariance at quadratic order. In fact the full leading Regge trajectory of QCD including daughter states seems to be reasonably well-described by fields which are dual to the twist two operators of QCD. It is also known from semi-classical arguments that operators of fixed twist receive corrections to their anomalous dimensions which are smaller than one would naively expect, growing only as the logarithm of the spin S [25]. Finally, we remind the reader of one other example where one finds remarkably accurate results without the presence of an obvious low-energy limit, namely level truncation in open string field theory [26].

We will have more to say about higher dimension terms in the effective action when we

construct the effective action for tensor fields in section 3. Whether or not these theoretical speculations are borne out, our fundamental justification for using a local 5d field theory approach is pragmatic. The best way to test any model is to extend it into new regimes, perform new calculations, and compare it with data.

The outline of this paper is as follows. In the second section we review basic facts about meson masses and decay constants in QCD as well as the quantum numbers of various quark bilinear operators. In the third section we introduce the fields dual to the dimension three tensor operators, and construct the free 5d Lagrangian. We then compute the spectrum and decay constants in the free theory. In section 4 we discuss interaction terms. There are a number of possible Lagrangians that one could consider and the proper choice requires a careful discussion of discrete symmetries. We also discuss the role of the $1/N_c$ expansion and dimensional analysis. In the final section we conclude and discuss some open issues. The appendices contain some useful technical details. In a subsequent paper we will present a detailed analysis of chiral symmetry breaking in this model, and the effect of the new interaction terms on the spectrum.

2 The QCD picture

We begin by describing the features of QCD that are most relevant to our analysis.

2.1 Operators and two-point functions

In QCD with two flavors the quark bilinear operators with naive dimension three are

$$\begin{aligned} O^{S,a}(x) &= \bar{q}(x)t^a q(x) \ , \\ O^{P,a}(x) &= \bar{q}(x)t^a\gamma_5 q(x) \ , \\ O^{V,a}_\mu(x) &= \bar{q}(x)t^a\gamma_\mu q(x) \ , \\ O^{A,a}_\mu(x) &= \bar{q}(x)t^a\gamma_\mu\gamma_5 q(x) \ , \\ O^{T,a}_{\mu\nu}(x) &= \bar{q}(x)t^a\sigma_{\mu\nu} q(x) \ , \end{aligned} \tag{2.1}$$

with $\sigma_{\mu\nu} = i(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2$. The index $a = 1,2,3$ for $SU(2)$ flavor symmetry or $a = 0,1,2,3$ for $U(2)$. In what follows we mostly suppress the flavor indices unless they are required for clarity. As the hard wall model of [7] contains fields dual to O^S, O^P, O^V, O^A, we emphasize here the role of the tensor operator O^T.

The operator O^T is odd under C but contains both parity even and odd parts and thus has a non-zero amplitude to create both $J^{PC} = 1^{--}$ and 1^{+-} states $\rho^{(n)}, b_1^{(n)}$:

$$\langle 0|O^{T,a}_{\mu\nu}(x)|b_1^{(n),c}(k)\rangle = \frac{i}{\sqrt{2}} f_b^{(n)} \epsilon_{\mu\nu\alpha\beta}\varepsilon^{(n)\alpha}_{(b)} k^\beta \delta^{ac} e^{-ik\cdot x} \ , \tag{2.2}$$

$$\langle 0|O^{T,a}_{\mu\nu}(x)|\rho^{(n),c}(k)\rangle = \frac{i}{\sqrt{2}} f_\rho^{T,(n)} (\varepsilon^{(n)}_{(\rho)\mu} k_\nu - \varepsilon^{(n)}_{(\rho)\nu} k_\mu)\delta^{ac} e^{-ik\cdot x} \ . \tag{2.3}$$

where $\varepsilon^{(n)}_{(\rho)}, \varepsilon^{(n)}_{(b)}$ denote transverse polarizations. In the rest frame of the mesons with $k = (m, \mathbf{0})$, (2.2) implies that the b_1 mesons are created by the transverse components O^T_{ij} while (2.3) implies that the ρ mesons are created by the longitudinal components O^T_{0i}. This

is consistent with the parity assignments of the b_1, ρ. Of course, ρ states are also created by the vector current[2]

$$\langle 0|O_\mu^{V,a}(x)|\rho^{(n),c}(k)\rangle = im_\rho^{(n)} f_\rho^{V,(n)} \varepsilon_{(\rho)\mu}^{(n)} \delta^{ac} e^{-ik\cdot x} \ . \tag{2.4}$$

We will see below that there are a number of ways to split up the degrees of freedom in O^T, and that different decompositions are practical in different contexts. For instance, we can project out the transverse and longitudinal parts of $O_{\mu\nu}^T$ in momentum space using the transverse and longitudinal projection operators $\mathcal{P}^\perp$, $\mathcal{P}^\parallel$ (see Appendix A for details) – these isolate the h_1/b_1 and ω/ρ-type states. We define $O^{T\perp}$ and $O^{T\parallel}$ via $(\mathcal{P}^\perp)_{\mu\nu}^{\alpha\beta} O_{\alpha\beta}^T = k^2 O_{\mu\nu}^{T\perp}$ and $(\mathcal{P}^\parallel)_{\mu\nu}^{\alpha\beta} O_{\alpha\beta}^T = k^2 O_{\mu\nu}^{T\parallel}$.

On the other hand, when we discuss the transformation properties of O^T under the chiral symmetry group $U(N_f)_L \times U(N_f)_R$ it is convenient to decompose the tensor into self-dual and anti-self-dual parts

$$O_{\mu\nu}^{T,\pm} = \bar{q}\sigma_{\mu\nu}\frac{1\pm\gamma_5}{2}q \ , \tag{2.5}$$

which obey

$$O_{\mu\nu}^{T,\pm} = \pm\frac{i}{2}\epsilon_{\mu\nu}{}^{\lambda\rho} O_{\lambda\rho}^{T,\pm} \ . \tag{2.6}$$

The operators $O^{T,\pm}$ transform as $(1,0)$ and $(0,1)$ under Lorentz transformations and obey $(O^{T,+})^* = O^{T,-}$. That is, complex conjugation flips the spacetime chirality, as is familiar from the behavior of Weyl spinors. Under the chiral symmetry group $O^{T,+}$ transforms as a bifundamental, $O^{T,+} \sim (\overline{\mathbf{N}}_f, \mathbf{N}_f)$ while $O^{T,-}$ transforms as the conjugate, $(\mathbf{N}_f, \overline{\mathbf{N}}_f)$.

We can also construct combinations of definite parity as

$$O_{\mu\nu}^T = O_{\mu\nu}^+ + O_{\mu\nu}^- \ , \tag{2.7}$$

which transforms as a tensor under parity and

$$O^{PT} = O_{\mu\nu}^+ - O_{\mu\nu}^- \ , \tag{2.8}$$

which transforms as a pseudotensor. These are not independent as a consequence of the identity $\sigma_{\mu\nu}\gamma_5 = i\epsilon_{\mu\nu\lambda\rho}\sigma^{\lambda\rho}$ which implies that

$$O_{\mu\nu}^{PT} = \frac{i}{2}\epsilon_{\mu\nu}{}^{\lambda\rho} O_{\lambda\rho}^T \ . \tag{2.9}$$

Our primary source for information about the properties of the dual theory will be the two-point function, whose momentum-space poles mark the masses of excitations, while the residues at the poles provide the decay constants. The large-Q behavior of holographic

[2] The literature contains many different conventions for the normalization of decay constants, some of which differ from those used here by factors of 2, $\sqrt{2}$ and powers of the meson mass. Our normalization for the decay constant f_ρ is fairly standard, our normalization for the decay constants f_b and f_ρ^T agrees with that used in [35, 36] once the difference in isospin conventions is taken into account.

correlators is often compared directly to QCD. We thus summarize the relevant QCD two-point functions here:

$$\begin{aligned}
\Pi^{V,V,ab}_{\mu\nu}(k) &= -i\int d^4x\; e^{ik\cdot x}\langle 0|O^{V,a}_{\mu}(x)O^{V,b}_{\nu}(0)|0\rangle \;, \\
\Pi^{T,V,ab}_{\lambda\rho,\mu}(k) &= -i\int d^4x\; e^{ik\cdot x}\langle 0|O^{T,a}_{\lambda\rho}(x)O^{V,b}_{\mu}(0)|0\rangle \;, \\
\Pi^{T,T,ab}_{\mu\nu,\lambda\rho}(k) &= -i\int d^4x\; e^{ik\cdot x}\langle 0|O^{T,a}_{\mu\nu}(x)O^{T,b}_{\lambda\rho}(0)|0\rangle \;.
\end{aligned} \tag{2.10}$$

It is conventional to separate out kinematical and group theory factors which are dictated by parity, current conservation and Lorentz invariance, so we define

$$\begin{aligned}
\Pi^{V,V,ab}_{\mu\nu}(k) &= (k^2\eta_{\mu\nu} - k_\mu k_\nu)\delta^{ab}\Pi^{V,V}(k^2) \;, \\
\Pi^{T,V,ab}_{\lambda\rho,\mu}(k) &= i(k_\lambda\eta_{\rho\mu} - k_\rho\eta_{\lambda\mu})\delta^{ab}\Pi^{T,V}(k^2) \;, \\
\Pi^{T,T,ab}_{\mu\nu,\lambda\rho}(k) &= \delta^{ab}\left(\mathcal{P}^{\parallel}_{\mu\nu,\lambda\rho}\Pi^{T,T\parallel}(k^2) + \mathcal{P}^{\perp}_{\mu\nu,\lambda\rho}\Pi^{T,T\perp}(k^2)\right) \;.
\end{aligned} \tag{2.11}$$

These two point functions receive both perturbative and nonperturbative contributions and have been studied in [27–31]. At large $-k^2$ these can be computed perturbatively in QCD and one finds

$$\begin{aligned}
&\Pi^{V,V}(k^2) \to \frac{N_c}{24\pi^2}\log(-k^2) \;, \\
&\Pi^{T,V}(k^2) \to -\frac{N_c}{4\pi^2}m_q\log(-k^2) \;, \\
&\Pi^{T,T\parallel}(k^2) \to \frac{N_c}{24\pi^2}\log(-k^2) \;, \qquad \Pi^{T,T\perp}(k^2) \to -\frac{N_c}{24\pi^2}\log(-k^2) \;,
\end{aligned} \tag{2.12}$$

where m_q is the quark mass. In the chiral limit, $m_q \to 0$, the tensor and vector currents do not mix perturbatively. This is consistent with the observation that O^T, O^V transform in different representations of the chiral symmetry group. Of course the perturbative mixing is proportional to the explicit chiral symmetry breaking parameter – but one should expect nonperturbative mixing, even in the chiral limit, due to the spontaneous chiral symmetry breaking of the quark condensate.

Given (2.2)-(2.4), it is clear that $\Pi^{V,V}$ and $\Pi^{T,T\parallel}$ should feature resonances corresponding to ω/ρ exchange, while $\Pi^{T,T\perp}$ should have resonances corresponding to h_1/b_1 exchange. By inserting a complete set of states into the two-point functions one learns

$$\begin{aligned}
&\Pi^{V,V}(k)|_{\text{poles}} = -\sum_n \frac{(f^{V,(n)}_\rho)^2}{k^2 - (m^{(n)}_\rho)^2} \;, \qquad \Pi^{T,T\parallel}(k)|_{\text{poles}} = -\sum_n \frac{(f^{T,(n)}_\rho)^2}{k^2 - (m^{(n)}_\rho)^2} \;, \\
&\Pi^{T,T\perp}(k)|_{\text{poles}} = \sum_n \frac{(f^{(n)}_b)^2}{k^2 - (m^{(n)}_b)^2} \;.
\end{aligned} \tag{2.13}$$

The decay "constants" here are the same as those appearing in (2.2)-(2.4). In general they run with scale.

2.2 Low-lying hadron spectrum and decay constants

As mentioned in the previous section, we can think of these vector and tensor operators as generating towers of increasingly massive spin-one resonances from the vacuum. The PDG summary table [32] lists three spin-one mesons with the quantum numbers of the isotriplet ρ meson: $\rho(770)$, $\rho(1450)$, and $\rho(1700)$ with masses in MeV of 775.49 ± 0.34, 1465 ± 25, and 1720 ± 20 respectively; and three spin one mesons with the quantum numbers of the isosinglet ω: $\omega(782)$, $\omega(1420)$, and $\omega(1650)$ with masses of 782.65 ± 0.12, $1400 - 1450$, and 1670 ± 30. Additional states such as the $\rho(1900)$, $\rho(2150)$ are mentioned in the complete particle listings and their mass spacing was quoted in [16] as part of the evidence for linearity in n of the mass squared of exited meson states m_n^2, but their existence must be regarded as uncertain. In comparison, the experimental evidence for the three lightest ρ states is now quite compelling [33].

quantity	exp/lattice result	source
m_{ρ^0}	775.49 ± 0.34	[32]
$m_{\rho'}$	1465 ± 25	[32]
$m_{\rho''}$	1720 ± 20	[32]
f_ρ	153 ± 7	[34]
$f_{\rho'}$	N/A	
$f_{\rho''}$	N/A	
f_ρ^T	184 ± 15	[36]
$f_{\rho'}^T$	N/A	
$f_{\rho''}^T$	N/A	
m_ω	872.65 ± 0.12	[32]
m'_ω	$1400 - 1450$	[32]
m_{b_1}	1229.5 ± 3.2	[32]
f_{b_1}	236 ± 23	[35]
m_{h_1}	1170 ± 20	[32]
m_{π^0}	134.9766 ± 0.0006	[32]
f_π	92.4 ± 0.35	[32]
m_{a_1}	1320 ± 40	[32]
f_{a_1}	433 ± 13	[32]
m_{f_1}	1281.8 ± 0.6	[32]

Table 1. Masses and decay constants of low-lying mesons in MeV. The decay constants f_{b_1}, f_ρ^T are evaluated at a scale of 2 GeV. Our focus is on 1^{--} and 1^{+-} states; the lightest axial-vectors and pseudoscalars are included for completeness.

We will denote the lowest three mass eigenstates by ρ, ρ' and ρ''. The corresponding vector and tensor decay constants, as defined by (2.2)-(2.4), will be denoted $f_\rho, f_{\rho'}, f_{\rho''}$ and $f_\rho^T, f_{\rho'}^T, f_{\rho''}^T$. In table 1 we summarize experimental results for masses and lattice results for decay constants of the tensor and vector mesons.

3 Holographic dual at the free level

Having reviewed the experimental and lattice data on the QCD side, we now extract this data from a holographic model. We first identify the field dual to the tensor operator O^T and its Lagrangian on the hard wall background. We then compute the masses and decay constants from the free equations of motion.

3.1 Field-operator correspondence and quadratic action

The gravitational background of the hard wall model consists of AdS_5 truncated at a finite radius [7]. Working with coordinates $x^M = (x^\mu, z)$ we use the metric

$$ds^2 = \frac{\ell^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2) \ , \qquad \varepsilon \le z \le z_0 \ , \tag{3.1}$$

where ℓ is the AdS radius and z_0 the infrared (IR) cutoff. (The AdS radius is often set to one, but we find it both useful and less confusing to keep it explicit). We will perform calculations at finite ε, taking the limit $\varepsilon \to 0$ at the end.

In what follows, e^{MNPQR} will denote the 5d Levi-Civita tensor with normalization $e^{0123z} = 1/\sqrt{g}$. We will often rewrite parts of the 5d action in terms of 4d fields; when we do so Greek indices $\mu, \nu, \cdots$ will be raised and lowered with the flat Minkowski metric $\eta_{\mu\nu}$. We also use $\epsilon^{\mu\nu\lambda\rho}$ for the 4d Levi-Civita symbol with $\epsilon^{0123} = 1$.

The hard wall model of [7] introduces the fields dual to the dimension three operators $O^{S,P}$ and to $O^{V,A}$ in the above background. These are, respectively, a tachyon field X in the bifundamental of the $U(N_f)_L \times U(N_f)_R$ gauge group, and the (axial)-vector gauge fields A, V. The fields live in a background geometry (3.1).

We now extend the hard wall model to include a field dual to the tensor operator O^T. The obvious choice is a two-form potential b_{MN}. To match the global flavor transformation properties of O^T, b_{MN} must transform as a bifundamental under the $U(N_f)_L \times U(N_f)_R$ gauge group, and must therefore be a complex field.

In order to match the six physical degrees of freedom generated by O^T (3 for each of the two massive vector-like states), b_{MN} should also carry a total of six degrees of freedom on shell. The complex field b_{MN} has twenty independent real components, but as will be seen explicitly later, the equations of motion imply a Proca-like condition which removes eight of these, leaving twelve independent on-shell degrees of freedom – twice as many as we want. In order to eliminate these additional states, one might consider imposing a tensor gauge invariance, but this turns out to be impossible. Since b transforms nontrivially under the gauge group, the usual three-form field strength $H = db$ is not gauge covariant. On the other hand, the gauge covariant antisymmetrized derivative

$$\mathcal{H}_{PMN} = 3D_{[P}b_{MN]} \tag{3.2}$$

is not invariant under tensor gauge transformations $\delta b_{MN} = 2\partial_{[M}\lambda_{N]}$ and as a result one cannot use tensor gauge invariance to remove the unphysical degrees of freedom.

This problem is familiar from the AdS/CFT literature where one also finds two-form tensor fields transforming under the $SO(6)$ gauge group in the dimensional reduction of

IIB string theory on S^5 [37, 38]. These fields are described by a first-order Lagrangian discussed in [39] and have been analyzed in the context of the AdS/CFT correspondence in [18, 19][3]. Though there is no tensor gauge invariance, none is needed since a first-order action has fewer independent degrees of freedom. In a first-order formalism half of these are momenta and half are coordinates, so one is left with six independent real degrees of freedom. This correctly matches the six physical degrees of freedom (two massive spin one vectors) created by the dual operator O^T.

According to the AdS/CFT map, a $d/2$-form field in AdS_{d+1} obeying a first-order equation with mass μ (in units of the AdS radius) is dual to an operator with dimension

$$\Delta = \frac{1}{2}(d + 2|\mu|) \ . \tag{3.3}$$

In contrast to the scaling dimension of the conserved current J_V, we should expect corrections to the naive value of $\Delta = 3$ for O^T. We thus leave μ arbitrary.

We now have the following extended version of the hard wall action:

$$S = S_{\rm hw} + S_{\rm CS} + S_{\rm sd} + S_{\rm int} \ , \tag{3.4}$$

where

$$S_{\rm hw} = \int d^5x\sqrt{g}\ \mathrm{tr}\ \left\{|DX|^2 + \frac{3}{\ell^2}|X|^2 - \frac{1}{4\ell g_5^2}(F_L^2 + F_R^2)\right\} \tag{3.5}$$

is the action used in [7], and

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int_{\mathcal{M}}\Big(\omega_5(A_L) - \omega_5(A_R)\Big) \ , \tag{3.6}$$

with $\mathrm{tr}F^3 = d\omega_5$, is the Chern-Simons term needed to match the flavor anomalies of QCD (see e.g. [41]), and

$$S_{\rm sd} = -\frac{i}{2\ell g_b^2}\int_{\mathcal{M}} \mathrm{tr}\ \left[\bar{b}\left(D - i\frac{\mu}{\ell}\star\right)b - b\left(D + i\frac{\mu}{\ell}\star\right)\bar{b}\right] - \frac{\mathrm{sgn}(\mu)}{4\ell g_b^2}\int_{\partial\mathcal{M}} \mathrm{tr}\ \bar{b}_{\mu\nu}b^{\mu\nu} \tag{3.7}$$

is the first order action for the antisymmetric tensor field written in terms of differential forms[4]. We use a bar to denote Hermitian conjugation on group indices–e.g. $\overline{X} \equiv X^\dagger$. Since we work in a basis of Hermitian generators, this amounts to complex conjugation of the individual flavor components. The interaction term $S_{\rm int}$ will be discussed later. Note that we have explicitly inserted factors of the AdS_5 radius, ℓ, in such a way that the couplings g_5 and g_b are dimensionless. The mass dimensions of the fields are $[X] = [b] = 3/2$, $[A_{L,R}] = 1$.

First order actions on manifolds with boundary require the addition of boundary terms as dictated by the requirement of a consistent variational principle [42] or from consistency

[3] At the level of free field theory one can establish an equivalence between the first order formalism and a second order formalism with a conventional kinetic energy term and a Chern-Simons mass term as in [40] but is is not clear that this equivalence can be established when interactions are included.

[4] We have $b = \frac{1}{2}b_{MN}dx^M dx^N$ and $Db \equiv \frac{1}{2}D_{[p}b_{MN]}dx^P dx^M dx^N$, and the Hodge dual is $(\star b)_{MNP} = \frac{1}{2}e_{MNP}{}^{QR}b_{QR}$. Since b is a bifundamental, like X, the gauge covariant derivative acts as $D_P b_{MN} = \partial_P b_{MN} - iA_{L,P}b_{MN} + ib_{MN}A_{R,P}$.

when passing between the Lagrangian and Hamiltonian formulations of the theory [43]. These terms play a crucial role in giving a precise definition to the AdS/CFT correspondence since the bulk action vanishes on the equations of motion, leaving the boundary term to generate correlation functions when we vary with respect to the sources.

The boundary term in (3.7) was first found by [18], but we can also obtain it via a variational argument as in [42]. Since this term will play an important role, we review the argument here. The boundary term is required for the variational problem to be well-defined: without the boundary term the variation of the action evaluated on shell will not vanish, implying that solutions to the equations of motion do not represent stationary points of the action, and thus invalidating the stationary phase approximation to the partition function.

We begin by analyzing the equations of motion to determine the appropriate degrees of freedom. Taking the variation of $S_{\rm sd}$ with respect to $\bar{b}$, the (free) equations of motion are

$$\left(d - i\frac{\mu}{\ell}\star\right) b = 0 \ . \tag{3.8}$$

We review the solution to these equations in some detail in Appendix B. Here we note simply that $b_{\mu z}$ is determined in terms of $b_{\mu\nu}$, and one may derive separate equations for the self-dual and anti-self-dual parts of $b_{\mu\nu}$. Writing

$$b_{\mu\nu}(x,z) = b^+_{\mu\nu}(x,z) + b^-_{\mu\nu}(x,z) \quad \text{with} \quad b^\pm_{\mu\nu} = \pm\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}b^{\pm\rho\sigma} \ , \tag{3.9}$$

the (anti) self-dual pieces obey the equations

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + k^2 - \frac{\mu(\mu\pm 2)}{z^2}\right] b^\pm_{\mu\nu}(k,z) = 0 \ , \tag{3.10}$$

where $b(k,z) = \int d^4x e^{ik\cdot x} b(x,z)$, as usual. Near the UV boundary $z \sim \varepsilon \to 0$, solutions to (3.10) behave as

$$b^+_{\mu\nu} \sim \tilde{S}_{\mu\nu}\varepsilon^{-\mu} - \tilde{s}_{\mu\nu}\varepsilon^{2+\mu} \ , \tag{3.11}$$

$$b^-_{\mu\nu} \sim \tilde{A}_{\mu\nu}\varepsilon^{2-\mu} - \tilde{a}_{\mu\nu}\varepsilon^{\mu} \ , \tag{3.12}$$

where $\tilde{S}, \tilde{s}$ and $\tilde{A}, \tilde{a}$ are self-dual and anti-self-dual polarizations, respectively. Since the equation of motion (3.8) is first order, however, these coefficients are not independent. One may derive the relation

$$(\tilde{A},\tilde{a})_{\mu\nu} = \frac{1}{k^2}\left((\mathcal{P}^\perp)^{\alpha\beta}_{\mu\nu} - (\mathcal{P}^\parallel)^{\alpha\beta}_{\mu\nu}\right)(\tilde{S},\tilde{s})_{\alpha\beta} \ . \tag{3.13}$$

Equivalently, we have

$$(\tilde{S},\tilde{s})_{\mu\nu} = \frac{1}{k^2}\left((\mathcal{P}^\perp)^{\alpha\beta}_{\mu\nu} - (\mathcal{P}^\parallel)^{\alpha\beta}_{\mu\nu}\right)(\tilde{A},\tilde{a})_{\alpha\beta} \ , \tag{3.14}$$

which is consistent with (3.13) since $(\mathcal{P}^\perp - \mathcal{P}^\parallel)^2 = k^4\mathbf{1}$. Observe that when $\mu > 0$, $\tilde{S}$ encodes the leading behavior of $b_{\mu\nu}$ near the UV boundary, while when $\mu < 0$, $\tilde{a}$ encodes the leading behavior. It will be important to distinguish these two cases below.

Now consider the variation of the bulk part of $S_{\rm sd}$, focusing on terms with z-derivatives:

$$\begin{aligned}\delta S^{\rm bulk}_{\rm sd} &= -\frac{i}{8\ell g_b^2}\delta\int_{\mathcal{M}} d^5x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{\bar{b}_{\mu\nu}\partial_z b_{\rho\sigma} - b_{\mu\nu}\partial_z\bar{b}_{\rho\sigma}\right\} + \cdots \\ &= -\frac{i}{8\ell g_b^2}\delta\int_{\mathcal{M}} d^5x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{\overline{b^+_{\mu\nu}}\partial_z b^-_{\rho\sigma} + \overline{b^-_{\mu\nu}}\partial_z b^+_{\rho\sigma} - b^+_{\mu\nu}\partial_z\overline{b^-_{\rho\sigma}} - b^-_{\mu\nu}\partial_z\overline{b^+_{\rho\sigma}}\right\} + \cdots \\ &= -\frac{i}{4\ell g_b^2}\int_{\mathcal{M}} d^5x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{\partial_z b^-_{\mu\nu}\delta\overline{b^+_{\rho\sigma}} + \partial_z b^+_{\mu\nu}\delta\overline{b^-_{\rho\sigma}} - \partial_z\overline{b^-_{\mu\nu}}\delta b^+_{\rho\sigma} - \partial_z\overline{b^+_{\mu\nu}}\delta b^-_{\rho\sigma}\right\} + \cdots \\ &\quad - \frac{i}{8\ell g_b^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{\overline{b^+_{\mu\nu}}\delta b^-_{\rho\sigma} + \overline{b^-_{\mu\nu}}\delta b^+_{\rho\sigma} - b^+_{\mu\nu}\delta\overline{b^-_{\rho\sigma}} - b^-_{\mu\nu}\delta\overline{b^+_{\rho\sigma}}\right\}^{z_0}_{\varepsilon}\ . \end{aligned} \tag{3.15}$$

Evaluating the variation on shell, the bulk terms vanish by the equations of motion. The relation (3.13) implies that we cannot simultaneously fix $b^+_{\mu\nu}$ and $b^-_{\mu\nu}$ at either the UV boundary or the IR boundary–to do so would overconstrain the system. At each boundary we may fix only one or the other. The natural choice at the UV boundary, from the AdS/CFT point of view, is to fix b^+ (b^-) for $\mu > 0$ ($\mu < 0$). (In the language of [18], for $\mu > 0$, b^+ plays the role of coordinate and b^- that of canonical momentum, while for $\mu < 0$ their roles are reversed). Either way, half of the boundary terms in the last line of (3.15) will remain. If we constrain b^+, say, then the δb^- terms will be nonzero.

There is a way out of this quandary. First let us suppose $\mu > 0$. Notice that the on-shell value of (3.15) may be written as

$$\begin{aligned}\delta S^{\rm bulk}_{\rm sd} = &-\frac{i}{8\ell g_b^2}\delta\int d^4x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{\overline{b^+_{\mu\nu}}b^-_{\rho\sigma} - b^+_{\mu\nu}\overline{b^-_{\rho\sigma}}\right\}^{z_0}_{\varepsilon} + \\ &+ \frac{i}{4\ell g_b^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{b^-_{\mu\nu}\delta\overline{b^+_{\rho\sigma}} - \overline{b^-_{\mu\nu}}\delta b^+_{\rho\sigma}\right\}^{z_0}_{\varepsilon}\ ,\end{aligned} \tag{3.16}$$

or, moving the first term to the left and using the (anti) self-duality of $b^\pm$,

$$\delta\left(S^{\rm bulk}_{\rm sd} - \frac{1}{4\ell g_b^2}\int_{\partial\mathcal{M}}{\rm tr}\left\{\bar{b}^{\mu\nu}b_{\mu\nu}\right\}\right) = \frac{i}{4\ell g_b^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{b^-_{\mu\nu}\delta\overline{b^+_{\rho\sigma}} - \overline{b^-_{\mu\nu}}\delta b^+_{\rho\sigma}\right\}^{z_0}_{\varepsilon}\ . \tag{3.17}$$

If $\mu < 0$ we should instead isolate δb^-. We find that the on-shell value of (3.15) may also be rewritten as

$$\delta\left(S^{\rm bulk}_{\rm sd} + \frac{1}{4\ell g_b^2}\int_{\partial\mathcal{M}}{\rm tr}\left\{\bar{b}^{\mu\nu}b_{\mu\nu}\right\}\right) = \frac{i}{4\ell g_b^2}\int d^4x\epsilon^{\mu\nu\rho\sigma}{\rm tr}\left\{b^+_{\mu\nu}\delta\overline{b^-_{\rho\sigma}} - \overline{b^+_{\mu\nu}}\delta b^-_{\rho\sigma}\right\}^{z_0}_{\varepsilon}\ . \tag{3.18}$$

Thus, by adding the boundary term, as has been done in (3.7), we allow for a consistent variational principle. The on-shell variation of the full (bulk + boundary) action will vanish if the boundary value of b^+ (b^-) is held fixed in the case $\mu > 0$ ($\mu < 0$).

We in fact have two choices for the boundary condition. Consider the $\mu > 0$ case for concreteness; analogous comments apply in the $\mu < 0$ case. When $\mu > 0$, the on-shell variation of (3.7) will vanish if we either hold b^+ fixed *or* set $b^- = 0$ on the boundary. We will refer to these as the Dirichlet-type and Neumann-type conditions, respectively, drawing on the coordinate and momentum characterization of $b^\pm$ given in [18]. On the UV boundary AdS/CFT dictates that we take the Dirichlet-type condition, holding b^+ fixed,

since b^+ represents the source for the dual operator. On the IR boundary, the correct choice is not immediately obvious.

The question is compounded by the potential presence of higher-order terms localized on the IR boundary and not included in (3.4). In general we should expect such terms to appear as a result of "integrating out" nontrivial IR dynamics up to the scale of the IR cutoff $1/z_0$. These terms could potentially modify the Neumann-type condition to a more general mixed condition, which may even be nonlinear, with parameters depending on the details of the localized terms[5].

These issues were dealt with in two different ways in the original hard wall model [7], depending on the dual field in question. In the case of the scalar field, X, all of these unknowns were packaged into a single parameter, the quark condensate, which was taken as an input that could be fit to data. In the case of the vector gauge field, V, the simple Neumann condition, $F_{z\mu}(z_0) \sim \partial_z V_\mu|_{z_0} = 0$, was chosen, which can be motivated as follows. First, one can definitively choose in favor of the Neumann condition over the Dirichlet one, $\delta V|_{z_0} = 0$, by requiring that the boundary condition be gauge invariant. This does not rule out the possibility of higher order terms in the field strength localized at the IR boundary, but the leading term of this sort was considered, and its effect was found to be small in practice. In general, one expects terms involving higher powers of the gauge field–on the boundary or in the bulk–to be suppressed by powers of $1/N_c$, while for X this is not true. We will review these arguments in section 4.

Returning to the case at hand, one may also use N_c counting to argue that higher order terms in b are suppressed. We do not have an analog of the gauge principle, however, to help us decide between the Neumann or Dirichlet-like condition. In the next subsection we use the holographic model (3.4) to compute the tensor-tensor two-point function, $\langle O^T O^T \rangle$, considering both IR boundary conditions. We will show that there is a strong physical argument for choosing the Neumann-like condition over the Dirichlet one: the Dirichlet-like condition leads to a massless divergence in the tensor-tensor two-point function, and there are no massless particles with the quantum numbers of the h_1/b_1-mesons in QCD! The Neumann-like condition, on the other hand, gives a sensible finite result for $\langle O^T O^T \rangle$ in the $k \to 0$ limit.

3.2 On-shell action and tensor-tensor two-point function

Let us now use the quadratic action (3.4), and in particular (3.7), to study the correlation functions and the normalizable spectrum of the complex two-form b_{MN}. The two-form b_{MN} corresponds to a total of six real degrees of freedom. We can package these in a variety of ways: in terms of real and imaginary, longitudinal and transverse, or self-dual and anti-dual parts of $b_{\mu\nu}$[6]. It will therefore be convenient to work as much as possible in terms

[5] We take the point of view that effects of higher order boundary terms modify the boundary conditions, such that a well defined variational principle is maintained. An alternative approach is to hold the simple Neumann-type boundary condition fixed, so the presence of higher-order terms leads to a contribution to the on-shell *value* of the action from the IR boundary. On a practical level one may take either point of view; these are just two different ways of encoding the effects of such higher-order terms.

[6] The components $b_{\mu z}$ can be eliminated through their equations of motion.

of projections onto these parts, $P^{\pm}$ and $\mathcal{P}^{\parallel,\perp}$, defined in Appendix A. The intermediate steps of the analysis differ between the cases $\mu > 0$ and $\mu < 0$, though the final result for the tensor-tensor two-point function can be given in a simple form, valid for both cases. In order to streamline the discussion in this subsection, we will present the $\mu > 0$ case only, and at the end quote the result for general μ. Details of the analysis are relegated to appendices B and C.

We first determine the bulk-to-boundary propagator of b, which we need in order to compute two-point functions of the dual operators.

For the free two-form in AdS_5 it is most convenient to work in terms of the self-dual piece which satisfies

$$b^+_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu}{}^{\rho\sigma} b^+_{\rho\sigma} \ . \tag{3.19}$$

As discussed by [18], it is the self-dual piece which sources the dual operator: for the value $\mu = 1$, which should give the scaling dimension in a *conformal* theory (i.e. on a background that is simply AdS_5), it indeed has the appropriate near-boundary scaling behavior to correspond to a tensor operator. In the confining theory, of course, the value of μ should receive quantum corrections.

The general solution to the free equations of motion for b is reviewed in Appendix B. In momentum space one has $b_{\mu\nu} = b^+_{\mu\nu} + b^-_{\mu\nu}$, with

$$\begin{aligned} b^+_{\mu\nu}(k,z) &= \tilde{S}_{\mu\nu}(k) z J_{-\mu-1}(kz) + \tilde{s}_{\mu\nu}(k) z J_{\mu+1}(kz) \ , \\ b^-_{\mu\nu}(k,z) &= \tilde{A}_{\mu\nu}(k) z J_{-\mu+1}(kz) + \tilde{a}_{\mu\nu}(k) z J_{\mu-1}(kz) \ , \end{aligned} \tag{3.20}$$

where the anti-self-dual polarizations are related to the self-dual ones by

$$\tilde{A}_{\mu\nu} = \frac{1}{k^2}(\mathcal{P}^{\perp} - \mathcal{P}^{\parallel})^{\alpha\beta}_{\mu\nu}\tilde{S}_{\alpha\beta} = \tilde{S}_{\mu\nu} - \frac{2}{k^2}\left(k_\mu k^\rho \tilde{S}_{\rho\nu} - k_\nu k^\rho \tilde{S}_{\rho\mu}\right) \ , \tag{3.21}$$

and similarly for $\tilde{a}$ in terms of $\tilde{s}$. Meanwhile, $b_{\mu z}$, which plays the role of Lagrange multiplier, is given by $b_{\mu z} = -\frac{z}{2\mu}\epsilon_\mu{}^{\nu\rho\sigma} k_\nu b_{\rho\sigma}$. The solution (3.20) is appropriate in the generic case of non-integer μ; if μ is an integer then the Bessel functions $J_{-\mu\mp1}$ should be replaced by $Y_{\mu\pm1}$.

We are interested specifically in the bulk-to-boundary propagator: that is, the solution to the equations of motion with the boundary condition that the self-dual field approaches a self-dual source on the UV boundary.

We first impose an IR boundary condition at $z = z_0$ to fix $\tilde{s}$ in terms of $\tilde{S}$. As we discussed above, there are two possibilities for a consistent variational principle: we may choose the Dirichlet-like condition where we hold δb^+ fixed, or the Neumann-like condition where we set $b^-_{\mu\nu}(z_0) = 0$. Holding b^+ fixed in this context means setting $b^+_{\mu\nu}(z_0) = 0$, since there is no natural constant antisymmetric two-tensor living on the IR boundary. After imposing one of these, we find that the solution takes the form

$$\begin{aligned} b^+_{\mu\nu}(k,z) &= \tilde{S}_{\mu\nu}(k)\left[zJ_{-\mu-1}(kz) - c^>_b(k,z_0) z J_{\mu+1}(kz)\right] \equiv \tilde{S}_{\mu\nu}(k) B^+_>(k,z) \ , \\ b^-_{\mu\nu}(k,z) &= \tilde{A}_{\mu\nu}(k)\left[zJ_{-\mu+1}(kz) - c^>_b(k,z_0) z J_{\mu-1}(kz)\right] \equiv \tilde{A}_{\mu\nu}(k) B^-_>(k,z) \ , \end{aligned} \tag{3.22}$$

where

$$c_b^>(k, z_0) = \begin{cases} \frac{J_{-\mu-1}(kz_0)}{J_{\mu+1}(kz_0)} \ , & b^+(z_0) = 0 \ , \\ \\ \frac{J_{-\mu+1}(kz_0)}{J_{\mu-1}(kz_0)} \ , & b^-(z_0) = 0 \ . \end{cases} \tag{3.23}$$

(The ">" labels are a reminder that these expressions are appropriate for the case $\mu > 0$).

The UV boundary condition fixes $\tilde{S}_{\mu\nu}$ in terms of the source for the dual operator. As we approach the UV boundary, $z = \varepsilon \to 0$, the leading scaling behavior of b is $b^+ \propto \varepsilon^{-\mu}$. We define the self-dual source, $S_{\mu\nu}(k)$, through

$$b^+_{\mu\nu}(k, \varepsilon) = \frac{\ell^{\mu-1/2}}{\varepsilon^\mu} S_{\mu\nu}(k) \ . \tag{3.24}$$

The factors of the AdS radius ℓ have been inserted on dimensional grounds. b is a dimension $3/2$ field, while S should have naive dimension 1 since it sources a naive dimension 3 operator. Using this boundary condition to eliminate $\tilde{S}$ in favor of the source S, we arrive at our final expression for the bulk to boundary propagator:

$$\begin{aligned} b^+_{\mu\nu}(k, z) &= \ell^{\mu-1/2} S_{\mu\nu}(k) \frac{B^+_>(k, z)}{\varepsilon^\mu B^+_>(k, \varepsilon)} \ , \\ b^-_{\mu\nu}(k, z) &= \ell^{\mu-1/2} \left[S_{\mu\nu} - \frac{2}{k^2} \left(k_\mu k^\rho S_{\rho\nu} - k_\nu k^\rho S_{\rho\mu} \right) \right] \frac{B^-_>(k, z)}{\varepsilon^\mu B^+_>(k, \varepsilon)} \ . \end{aligned} \tag{3.25}$$

Pulling out the explicit factor of $\varepsilon^{-\mu}$, as we have done in (3.24), ensures that our final expressions for two-point functions will be ε-independent and corresponds to working with "rescaled" operators. Another procedure commonly appearing in the literature is to simply set $\ell^{1/2} b^+(k, \varepsilon) = S(k)$, in which case our result for the two-point function would include an overall factor of $\varepsilon^{2\mu}$, indicating its scaling behavior as one approaches the UV boundary. In the $\varepsilon \to 0$ limit one should trade this factor for a renormalization scale M_r. In (3.24) we have chosen to identify the renormalization scale with the AdS radius, $M_r \sim \ell^{-1}$. This is natural since the dimensionful couplings $\ell^{1/2} g_5$, $\ell^{1/2} g_b$ appearing in (3.4) were already (implicitly) defined at this scale.

To summarize, three different length scales appear in the model: the locations of the UV and IR boundaries at ε and z_0, respectively, and ℓ the AdS radius. In terms of the dual field theory, the mass scale ε^{-1} is the UV cutoff we would include in the computation of bare n-point functions. Adding the appropriate counterterms to the action is essentially equivalent to replacing ε^{-1} with ℓ^{-1}, to give a finite (renormalized) result. ℓ^{-1} is the renormalization scale, while z_0^{-1} is $\Lambda_{\rm QCD}$.

These statements may seem trivial when we are working with a simplified model consisting of AdS_5 with cutoff. While this model captures basic features such as confinement, couplings and dimensions of operators do not run with scale. In a more realistic model this running would be encoded in a nontrivial z-dependent geometry. Physically, the picture is that the asymptotic part of our AdS slice is not describing asymptotically free QCD, but rather is providing an approximate description in a window of scales where the QCD

coupling is finite, but running slowly with scale. We have already stressed that the tensor operator O^T is not a conserved current, and at strong coupling one should expect $\mathcal{O}(1)$ corrections to its charge and dimension – or to g_b, μ in the dual language. The renormalization scale ℓ^{-1} represents a typical scale in this window. The ratio of this scale to the QCD scale, parameterized by the ratio z_0/ℓ, is a dimensionless parameter in our model, and we will see that it naturally appears in physical quantities like decay constants.

To compute correlators according to the usual AdS/CFT description, we evaluate the action on the solution (3.25), functionally differentiate with respect to the source, and then take the $\varepsilon \to 0$ limit. $S_{\mu\nu}$ couples to the self-dual operator, $O^{T,+}_{\mu\nu}$ in (2.5). It is related to the source, $\mathcal{T}_{\mu\nu}$, for the tensor operator O^T via

$$S_{\mu\nu} = \mathcal{T}_{\mu\nu} + \frac{i}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\mathcal{T}_{\rho\sigma} \quad \Rightarrow \quad \mathcal{T}_{\mu\nu}(k) = \frac{1}{2}\left(S_{\mu\nu}(k) + \bar{S}_{\mu\nu}(-k)\right) \ . \tag{3.26}$$

The sole contribution to the on-shell action comes from the UV boundary term, which we find to be

$$S_{\rm sd} = \frac{\ell^{2\mu-2}}{g_b^2}\int \frac{d^4k}{(2\pi)^4}\frac{B^-_>(k,\varepsilon)}{k^2\varepsilon^{2\mu}B^+_>(k,\varepsilon)}\mathrm{tr}\left\{\mathcal{T}^{\alpha\beta}(-k)\left[\mathcal{P}^{\perp}_{\alpha\beta,\delta\gamma} - \mathcal{P}^{\parallel}_{\alpha\beta,\delta\gamma}\right]\mathcal{T}^{\delta\gamma}(k)\right\} \ . \tag{3.27}$$

This action leads to the following matrix elements. We have that $\Pi^{T,T\parallel} = -\Pi^{T,T\perp}$, as is evident from (3.27), and

$$\Pi^{T,T\perp}(k) = \begin{cases} -\frac{\Gamma(-\mu)}{2^{2\mu-2}g_b^2\Gamma(\mu)}c_b^>(k,z_0)(k\ell)^{2\mu-2} \ , & \mu > 0 \text{ non-integer} \ , \\ \\ \frac{1}{2^{2\mu-2}g_b^2\mu!(\mu-1)!}\left[\pi c_b^>(k,z_0) - \log\left(k^2\ell^2\right)\right](k\ell)^{2\mu-2} \ , & \mu > 0 \text{ integer} \ , \end{cases} \tag{3.28}$$

where $c_b^>(k,z_0)$ is given by (3.23) in the non-integer μ case and by (3.23) with $J_{-\mu\mp1} \to Y_{\mu\pm1}$ in the integer μ case. Some of the intermediate steps involved in obtaining (3.27), (3.28), are presented in Appendix C.

Let us consider the extreme IR limit of this result, $k \to 0$. The tensor-tensor correlator should be finite in this limit, since there would be no particle interpretation in QCD for such a massless divergence. The leading k^2 behavior of $c_b^>(k,z_0)$ is (for both integer and non-integer μ)

$$\lim_{k\to0} c_b^>(k,z_0) = \begin{cases} \mathcal{O}(k^{-2\mu-2}) \ , & b^+(z_0) = 0 \ , \\ \mathcal{O}(k^{-2\mu+2}) \ , & b^-(z_0) = 0 \ . \end{cases} \tag{3.29}$$

We conclude that the Dirichlet-like boundary condition, $b^+(z_0) = 0$, is physically unacceptable since it leads to a divergence in $\Pi^{T,T}$ at large distances. The Neumann-like condition, $b^-(z_0) = 0$, on the other hand gives a physically reasonable result for the $k \to 0$ limit of $\Pi^{T,T}$. Per our discussion in the previous subsection, we take the Neumann-like boundary condition to be our IR condition for the b-sector.

The final result for the tensor-tensor two-point function, in the case $\mu > 0$, computed using the free dual (3.7), is

$$\Pi^{T,T\perp}(k) = \begin{cases} -\frac{\Gamma(-\mu)}{2^{2\mu-2}g_b^2\Gamma(\mu)}(k\ell)^{2\mu-2}\frac{J_{-\mu+1}(kz_0)}{J_{\mu-1}(k,z_0)} \ , & \mu > 0 \text{ non-integer} \ , \\ \\ \frac{1}{2^{2\mu-2}g_b^2\mu!(\mu-1)!}(k\ell)^{2\mu-2}\left[\pi\frac{Y_{\mu-1}(kz_0)}{J_{\mu-1}(kz_0)} - \log\left(k^2\ell^2\right)\right] \ , & \mu > 0 \text{ integer} \ . \end{cases} \tag{3.30}$$

In appendices B and C we also analyze the $\mu < 0$ case. The final result is identical to (3.30) with $\mu \to |\mu|$, up to a sign whose origin may be traced to the sign on the boundary term in (3.7). Thus the general result, valid for any $\mu \neq 0$, is

$$\Pi^{T,T\perp}(k) = \begin{cases} -\frac{\text{sgn}(\mu)\Gamma(-|\mu|)}{2^{2|\mu|-2}g_b^2\Gamma(|\mu|)}(k\ell)^{2|\mu|-2}\frac{J_{-|\mu|+1}(kz_0)}{J_{|\mu|-1}(kz_0)} \ , & \mu \text{ non-integer} \ , \\ \\ \frac{\text{sgn}(\mu)}{2^{2|\mu|-2}g_b^2|\mu|!(|\mu|-1)!}(k\ell)^{2|\mu|-2}\left[\pi\frac{Y_{|\mu|-1}(kz_0)}{J_{|\mu|-1}(kz_0)} - \log\left(k^2\ell^2\right)\right] \ , & \mu \text{ integer} \ . \end{cases} \tag{3.31}$$

Next we turn to a discussion of the physics contained in (3.31).

3.3 Interpretation of results

The original hard-wall model depends on the parameters z_0, m_q, and σ, as well as g_5 which is expressed in units of the *AdS* radius ℓ. Our posited extension of the hard-wall model contains new parameters μ, g_b as well as explicit dependence on ℓ. All physical quantities we compute will be given in terms of these parameters, so in order to make additional predictions, we need to fix them using real QCD results. Of course it is best, when possible, to fix these parameters exactly using explicit computations in QCD. For example, quantities which are not renormalized, such as the anomalous dimension of a conserved current, give the same result at weak or strong coupling. The authors of [7] successfully employed this strategy to find g_5 in (3.4), by comparing the gravity dual and perturbative QCD results for the large Q^2 J_V-J_V correlator. Naively, one might want to fix μ and g_b by comparing the UV (large momentum) limit of the tensor two-point functions to large-momentum QCD. While this method is justified for finding g_5 using the two-point function of the *conserved* current J_V, the operator O^T is not conserved, so we have no reason to expect that μ and g_b will not receive significant corrections at strong coupling. (Incidentally, the issue applies to the scaling dimension of $\bar{q}q$, dual to the 5d mass of the tachyon field X which was held at its naive value in [7]).

The two-point function (3.31) encapsulates much of the new information we obtain from adding the non-interacting two-form field: the locations of its k-space poles mark the masses of mesonic resonances generated by O^T while the residues at the poles give the corresponding decay constants. We can expand the two-point function around its poles, and by comparison to (2.13), read off these quantities[7].

[7] This is equivalent to identifying the masses as the eigenvalues of normalizable modes and the decay

By inspection of (3.31) we see that the two-point function (for both integer and non-integer μ) has an infinite set of simple poles for values of $k = m$ such that $J_{|\mu|-1}(mz_0) = 0$: these define the masses of the states created by O^T to be $m_n = x_{|\mu|-1,n} z_0^{-1}$ where $x_{|\mu|-1,n}$ denotes the n-th zero of $J_{|\mu|-1}$. Note that each pole corresponds to two degenerate states: a 1^{+-} (h_1/b_1-like) state and a 1^{--} (ω/ρ-like) state.

Taylor-expanding near the first pole, we find

$$\Pi^{T,T\perp} = \begin{cases} -\frac{\mathrm{sgn}(\mu)}{g_b^2}\left(\frac{x_{|\mu|-1,1}\ell}{2z_0}\right)^{2|\mu|-2} \frac{2\Gamma(-|\mu|)J_{1-|\mu|}(x_{|\mu|-1,1})}{x_{|\mu|-1,1}\Gamma(|\mu|)J'_{|\mu|-1}(x_{|\mu|-1,1})} \frac{m_1^2}{k^2-m_1^2} + \cdots \ , \\ \qquad\qquad \mu \text{ non-integer}, \\ \frac{\mathrm{sgn}(\mu)}{g_b^2}\left(\frac{x_{|\mu|-1,1}\ell}{2z_0}\right)^{2|\mu|-2} \frac{2\pi Y_{|\mu|-1}(x_{|\mu|-1,1})}{|\mu|!(|\mu|-1)!x_{|\mu|-1,1}J'_{|\mu|-1}(x_{|\mu|-1,1})} \frac{m_1^2}{k^2-m_1^2} + \cdots \ , \\ \qquad\qquad \mu \text{ integer}, \end{cases} \tag{3.32}$$

where J' is the derivative of the Bessel function with respect to its argument. On comparing this with (2.13), we learn that μ should be taken negative, and that the decay constant for the first resonances has the form

$$f_b^{(1)}(m_1) = \begin{cases} \frac{1}{g_b}\left(\frac{\ell}{z_0}\right)^{|\mu|-1}\left[\left(\frac{x_{|\mu|-1,1}}{2}\right)^{|\mu|-1}\sqrt{\frac{2x_{|\mu|-1,1}\Gamma(-|\mu|)J_{1-|\mu|}(x_{|\mu|-1,1})}{\Gamma(|\mu|)J'_{|\mu|-1}(x_{|\mu|-1,1})}}\right] z_0^{-1} \ , \\ \qquad\qquad \mu \text{ non-integer}, \\ \frac{1}{g_b}\left(\frac{\ell}{z_0}\right)^{|\mu|-1}\left[\left(\frac{x_{|\mu|-1,1}}{2}\right)^{|\mu|-1}\sqrt{-\frac{2x_{|\mu|-1,1}\pi Y_{|\mu|-1}(x_{|\mu|-1,1})}{|\mu|!(|\mu|-1)!J'_{|\mu|-1}(x_{|\mu|-1,1})}}\right] z_0^{-1} \ , \\ \qquad\qquad \mu \text{ integer}. \end{cases} \tag{3.33}$$

We emphasize that $f_b^{(1)}(k)$ depends on the momentum, and that the prediction made here is for the on-shell value of the first resonance. Note also that the sign of μ turns out to be significant: we must choose one sign over the other to have real decay constants[8].

Based on the non-interacting model we have described so far, we were able to make concrete predictions for the masses and decay constants of the ω'/ρ' and the h_1/b_1 meson in terms of the parameters μ, g_b, and z_0/ℓ. Note that if we set $|\mu| = 1$, its value in perturbative QCD, we would find that both the ω'/ρ' and the h_1/b_1 states are degenerate with the ω/ρ mesons generated by the vector current, whose experimental mass is approximately half of the ω'/ρ' mass!

Conversely, we can use the measured value of the h_1/b_1 mass of $m = 1230$ MeV to estimate that $|\mu|$ runs to ≈ 1.82 at this scale, with

$$f_b^{(1)}(m_1) \approx \frac{1}{g_b}\left(\frac{\ell}{z_0}\right)^{0.82} 2092 \text{ MeV} \ . \tag{3.34}$$

constants as derivatives of the eigenfunctions. Especially when one studies interactions it is often useful to work with the normalizable eigenfunctions, and to write down an effective 4d action for the corresponding resonances by integrating out the holographic z direction. We consider this point of view in Appendix D, but here it is equally convenient to just work with the two-point function.

[8]The fact that μ turns out to be negative – which implies that we are fixing the value of an anti-self-dual source on the UV boundary – has no physical meaning, however: if we had instead chosen $\bar{b}$ as our fundamental field and performed the entire analysis above for $\bar{b}$ instead of b, we would have instead found that we had to fix a self-dual source on the UV boundary.

While we could use this data to now fix the value of another parameter, we see that our description of the tensor operator is still incomplete. So far we have introduced O^T with a non-interacting Lagrangian, reading off the masses and decay constants of the resonances it creates; these particles are totally ignorant of chiral symmetry breaking, and furthermore have no decay modes. In order to make better contact with QCD, lifting the degeneracy between the 1^{+-} and 1^{--} states and introducing interactions that allow the vector- and tensor-generated ω/ρ mesons to mix, we must add 5d interaction terms to the bulk Lagrangian. We will now discuss how to identify these terms.

4 Interactions: discrete symmetries and N_c counting

In order to classify the modes of b_{MN} corresponding to various meson states, we must carefully understand the holographic equivalents of discrete symmetries in QCD. Invariance of the gravity dual action under these symmetries will also constrain the interactions we can legally include in $S_{\rm int}$.

Consider the pure gauge action, ignoring the Chern-Simons terms:

$$S_{\rm gauge} = -\frac{1}{4\ell g_5^2}\int d^5x\sqrt{g}\,{\rm tr}\Big(F_L^2 + F_R^2\Big) \ . \tag{4.1}$$

This sector of the theory has five Z_2 symmetries. One of these is T, time reversal invariance, which will not be needed in what follows. There is also five-dimensional parity, which acts to reverse the sign of an odd number of the spatial coordinates; we take its action to be

$$P_5 : A^M(x^0,\vec{x},z) \to (-1)^{I(M)}A^M(x^0,-\vec{x},z) \ , \tag{4.2}$$

where $I(M)$ is 0 if $M = 0, z$ and 1 if $M = 1, 2, 3$. In addition there are three Z_2 symmetries which do not act on the coordinates:

$$\begin{aligned} \tilde{P} &: A_L \leftrightarrow A_R \ , \\ C_L &: A_L \to -A_L^* \ , \\ C_R &: A_R \to -A_R^* \ . \end{aligned} \tag{4.3}$$

Regarding the last two, we write the Lie algebra of the gauge group as

$$[T^a, T^b] = i f^{abc} T^c \ , \tag{4.4}$$

with the T^a Hermitian, normalized to ${\rm tr}(T^aT^b) = \delta^{ab}/2$ and the structure constants f^{abc} real. It is then easy to check that $-(T^a)^*$ obey the same Lie algebra as the T^a so the transformation $T^a \to -(T^a)^*$ is an automorphism of the Lie algebra. For $SU(N)$ this transformation takes the generators in the $\mathbf{N}$ to those in the $\overline{\mathbf{N}}$. For $U(1)$ it just takes $A_\mu \to -A_\mu$ which is the usual action of C on the vector potential, so it makes sense to call such a transformation charge conjugation. The action $S_{\rm gauge}$ has separate charge conjugation symmetries for $U(N_f)_L$ and $U(N_f)_R$.

We now consider adding the Chern-Simons term

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int_{\mathcal{M}}\Big(\omega_5(A_L) - \omega_5(A_R)\Big) \ . \tag{4.5}$$

This term changes sign under P_5 which changes the orientation of $\mathcal{M}$ and it also clearly changes sign under $\tilde{P}$. Thus this term is only invariant under the combination $P \equiv P_5\tilde{P}$. This is analogous to the discussion in [44] of Z_2 symmetries in the pion low-energy effective action where one finds an extra Z_2 symmetry of the action which is not a symmetry of QCD, and which is only broken by the addition of the Wess-Zumino-Witten term.

Since $\mathrm{tr}F_L^3 = d\omega_5(A_L)$ and under C_L we have $\mathrm{tr}F_L^3 \to -\mathrm{tr}F_L^T F_L^T F_L^T = -\mathrm{tr}F_L^3$ we see that $\omega_5(A_L)$, $\omega_5(A_R)$ are odd under C_L, C_R. Thus the Chern-Simons term is invariant under the combinations $C_L C_R \tilde{P}$, or under $C_L C_R P$. We will pick the first one and call it C, $C \equiv C_L C_R \tilde{P}$. Thus the pure gauge action, including the Chern-Simons term, is invariant under a $Z_2 \times Z_2$ symmetry generated by C and P.

We now extend these symmetries to the bifundamental fields X, b. The covariant derivative term $|DX|^2$ is invariant provided that $C : X \to \pm X^T, P : X \to \pm X^\dagger \equiv \pm\overline{X}$, but since changing the sign of X can be accomplished by a gauge transformation we can choose the action to be

$$\begin{aligned} &C : X \to X^T \ , \\ &P : \ X \to \overline{X} \ . \end{aligned} \tag{4.6}$$

where the action of P on the argument of X is the same as P_5 in (4.2) and will be suppressed from now on. Note that if we follow the treatment in [7] and expand X around its vacuum expectation value as $X = X_0(z)\exp(2i\pi^a T^a)$ then the above action of C, P agrees with the canonical assignments of C, P to the pion fields[9]. The terms in the action involving b are invariant under C, P provided that

$$\begin{aligned} &C : b_{MN} \to \pm b_{MN}^T \ , \\ &P : b_{MN} \to \pm(-1)^{I(M,N)}\bar{b}_{MN} \ . \end{aligned} \tag{4.7}$$

where $I(M, N)$ is the number of (i, j) indices in (MN). We can fix the sign in the action of C by comparison to the standard assignment of $C = -1$ to the h_1 meson. The action of P on the real and imaginary parts of b_{MN} differs by a sign, so the choice of sign in the action of P just changes which fields we associate to the real and imaginary parts of b_{MN}. In what follows we choose the plus sign so that

$$\begin{aligned} &C : b_{MN} \to -b_{MN}^T \ , \\ &P : b_{MN} \to (-1)^{I(M,N)}\bar{b}_{MN} \ . \end{aligned} \tag{4.8}$$

Any bottom-up 5d dual contains, in principle, an infinite number of possible interaction terms. Such terms are not only relevant to higher point functions between QCD operators: when $\bar{q}q$ acquires a vev, breaking chiral symmetry, these interactions also contribute to two-point functions, modifying the location of the poles and values of the decay constants. In the original hard wall model [7], this mechanism breaks the degeneracy between vector and axial-vector modes. Once we include b_{MN}, such terms will induce the mixing between vector meson states generated by O^T, and those generated by O^V.

[9]Later on we will find it more convenient to make a field redefinition which embeds the 4d pion field entirely in the axial-vector gauge field.

It is not trivial to decide which of this infinite set of terms to include in our analysis. In the language of effective field theory we often group interactions by dimension and assume that higher dimension operators are suppressed by some scale. From the action in (3.4) we find that the mass dimensions of the fields are

$$[X] = 3/2 \ , \qquad [b] = 3/2 \ , \qquad [F] = 2 \ . \tag{4.9}$$

so in principle we should only consider interaction terms, such as those roughly of the form bFX, which have dimension 5. No real separation of scales exists in the hard wall model, however, so it is not clear that this classification of dominant interaction terms is valid.

Another possible organizing principle is to consider instead the large-N_c scaling of interaction terms, and to include only those leading in $1/N_c$. We first recall the standard large N_c counting rules for QCD as reviewed in [45, 46]. Consider quark bilinear operators, $\hat{H}_i$, which can include any number of gauge fields as well. The large N_c rules for QCD diagrams give

$$\langle \hat{H}_1 \cdots \hat{H}_r \rangle \sim N_c^{1-r} \ . \tag{4.10}$$

The operators $\hat{H}_i$ are normalized so that $\sqrt{N_c}\hat{H}_i$ has an amplitude of order one to create a one meson state. Therefore matrix elements of operators that create mesons with unit amplitude behave as

$$\langle \sqrt{N_c}\hat{H}_1 \cdots \sqrt{N_c}\hat{H}_r \rangle \sim N_c^{1-r/2} \ . \tag{4.11}$$

The two-point function is $O(1)$ and the three-point function is $O(1/\sqrt{N_c})$. This shows that the coupling is $O(1/\sqrt{N_c})$, and a two-body decay amplitude for a meson is $O(1/\sqrt{N_c})$ so mesons become stable in the large N_c limit.

In the hard wall model we have been using a Lagrangian schematically of the form

$$S \sim \frac{1}{g_5^2} \int F^2 + \frac{1}{g_b^2} \int b^2 + \int (|DX|^2 + |X|^2) + \cdots \ , \tag{4.12}$$

where $\cdots$ represents additional interaction terms. If we define rescaled gauge fields A' via $A = g_5 A'$ and a rescaled tensor field via $b = g_b b'$ then we have

$$S \sim \int F'^2 + \int b'^2 + \int (|DX|^2 + |X|^2) + \cdots \ . \tag{4.13}$$

By computing two- and three-point functions and matching to the N_c counting of the field theory, one finds $g_5 \sim 1/\sqrt{N_c}$ which is consistent with the result found in [7] from matching the UV behavior of the vector current two-point function. The field b only appears quadratically in what we have done so far, so the N_c dependence of g_b is not determined. If we use the above Lagrangian to compute two-point functions by the usual prescription in AdS/CFT they will be order N_c^0 in agreement with (4.11). Three-point functions, which involve couplings between three gauge fields, or a gauge field and two pion fields and so on, will involve a factor of g_5 from the rescaling, and so will indeed be of order $1/\sqrt{N_c}$.

Terms involving the tachyon field X are more subtle. Consider for example a coupling of the form bXF which can contribute to two-point functions when X is set equal to its vev, and apparently also to three-point couplings when we include fluctuations of X

(pion modes). If we only focus on pion couplings then we can write $X = X_0 e^{2i\pi}$ which we leave invariant while redefining the gauge fields and b field by what would be a gauge transformation if we were transforming X as well and with $V_L = V_R^\dagger = e^{i\pi}$. This removes the pion field from X and puts it entirely in the axial gauge field. After this redefinition, we can replace any X appearing in the action with X_0. Now the bXF term contributes only to the two-point function, and one might conclude that its coupling should be of order N_c^0. However in general one could also look at fluctuations in the magnitude of X which are dual to the broad σ resonance of QCD. This term then contributes to three-point couplings involving σ, a vector meson and a h_1/b_1 meson and one concludes that in fact its coupling should be of order $1/\sqrt{N_c}$. This term thus gives a subleading contribution to the two-point function in the $1/N_c$ expansion. Terms containing even more powers of X will be suppressed by additional powers of $1/\sqrt{N_c}$ and should be small at large N_c.

We now consider two interaction terms that are consistent with all global and local symmetries and are the leading terms that lead to tensor-vector mixing in the ω/ρ sector, and break the degeneracy between the h_1/b_1 and tensor-ω/ρ spectrum. While conceptually straightforward, a thorough analysis of the interacting model turns out to be technically complex, and is postponed to a forthcoming paper [47].

The first term we consider is the unique cubic, dimension five operator which is P, C and $U(N_f)_L \times U(N_f)_R$ invariant:

$$S_{g_1} = g_1 \int d^5x \sqrt{g} \mathrm{tr} \left\{ b_{MN} F_R^{MN} \overline{X} + \overline{X} F_L^{MN} b_{MN} + X F_R^{MN} \bar{b}_{MN} + \bar{b}_{MN} F_L^{MN} X \right\} \ . \tag{4.14}$$

By the earlier argument after rescaling fields we find $g_1 g_b g_5 \sim O(N_c^{-1/2})$ which implies $g_1 g_b \sim O(N_c^0)$. Evaluating X on its chiral symmetry breaking vev, $\langle X \rangle = \langle \overline{X} \rangle \propto \mathbf{1} v(z)$, one finds a quadratic order term that mixes the vector gauge field V with the two-form b. The mixing modifies the mass spectrum of both vector and tensor-ω/ρ states, and breaks the degeneracy between the spectrum of h_1/b_1 and tensor-ω/ρ states.

The second interaction term we consider is the dimension six term

$$S_{g_2} = g_2 \ell \int d^5x \sqrt{g} \mathrm{tr} \left\{ b_{MN} \overline{X} b^{MN} \overline{X} + \bar{b}_{MN} X \bar{b}^{MN} X \right\} \ , \tag{4.15}$$

and we have $g_2 g_b \sim O(N_c^{-1/2})$. Although this term does not contribute directly to tensor-vector mixing in the ω/ρ sector, it does contain quadratic terms, when X is evaluated on its vev, that break the degeneracy between the h_1/b_1 and tensor-ω/ρ sectors.

In a follow-up paper we will analyze the interacting model (3.4), with $S_{\mathrm{int}} = S_{g_1} + S_{g_2}$.

5 Conclusions

We have presented a consistent formalism for including fields dual to the antisymmetric tensor quark bilinear O^T in a five-dimensional gravity dual description of QCD. This operator creates $J^{PC} = 1^{+-}$ mesons such as the h_1 and b_1 mesons as well as $J^{PC} = 1^{--}$ ω/ρ-like mesons.

While the principle of including this new field in the holographic framework is relatively straightforward, the implementation required several novel strategies. One was the

use of a first order action, required for gauge invariance under $U(N_f)_L \times U(N_f)_R$ flavor transformations while demanding that the fields give rise to the correct number of degrees of freedom in four dimensions. Another key technical point was to determine a unique IR boundary condition for the new field, which was based on the physical requirement that O^T should not generate a zero-momentum mode. We should also note that for O^T we are forced to account for the running of the anomalous dimension, rather than simply fitting to the free field dimension as suffices for mesons created by conserved currents.

We also performed an analysis of the interaction terms allowed by the discrete and gauge symmetries of the model, and classified the lowest dimension and leading order in $1/N_c$ terms that mix different states with the quantum numbers of the ρ, ω mesons in the presence of chiral symmetry breaking. In principle this model can now make new predictions for a number of physical quantities including

- the mass spectrum and decay constants for the h_1, b_1 mesons,
- the rates for $h_1 \to \rho + \pi$ and $b_1 \to \omega + \pi$ as well as the D/S amplitude ratio for these decays,
- the mass spectrum and vector and tensor decay constants for the low-lying excited ρ and ω meson states.

Unfortunately this analysis requires rather complicated numerical analysis and as a result we postpone presentation of these results to a later paper [47].

Finally, we reiterate that although we have focused on the hard wall model, our considerations apply more generally to a broad class of dual models of QCD. The soft wall model and other models that modify the dilaton and/or metric require antisymmetric tensor fields of the type described here in order to incorporate h_1/b_1 mesons. It would also be interesting to see whether the action described here for these fields can be derived for top-down models such as the Sakai-Sugimoto model using the techniques of open string field theory.

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A Projection operators: definitions and useful relations

We will often find it useful to consider the (anti) self-dual or longitudinal/transverse pieces of the two-form $b_{\mu\nu}$. We can define these in terms of the operators $P^{\pm}$ and $\mathcal{P}^{\|,\perp}$:

$$
\begin{aligned}
(\mathcal{P}^{\|})^{\mu\nu}_{\alpha\beta} &= 2k_{[\alpha}k^{[\mu}\delta^{\nu]}_{\beta]} \ , \\
(\mathcal{P}^{\perp})^{\mu\nu}_{\alpha\beta} &= (k^2\mathbf{1} - \mathcal{P}^{\|})^{\mu\nu}_{\alpha\beta} = k^2\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta]} - 2k_{[\alpha}k^{[\mu}\delta^{\nu]}_{\beta]} \ , \\
(P^{\pm})^{\mu\nu}_{\alpha\beta} &= \frac{1}{2}\left(\delta^{\mu}_{[\alpha}\delta^{\nu}_{\beta]} \pm \frac{i}{2}\epsilon_{\alpha\beta}{}^{\mu\nu}\right) \ .
\end{aligned} \tag{A.1}
$$

Useful relations among the projectors include

$$
P^{\pm}\mathcal{P}^{\perp}P^{\pm} = \frac{k^2}{2}P^{\pm} \ , \qquad P^{\pm}\mathcal{P}^{\perp}P^{\mp} = \frac{1}{2}(\mathcal{P}^{\perp} - \mathcal{P}^{\|})P^{\mp} \ . \tag{A.2}
$$

The projectors $P^{\pm}$ are idempotent, $P^2 = P$, while the projectors $\mathcal{P}^{\|,\perp}$ have a non-standard normalization: $\mathcal{P}^2 = k^2\mathcal{P}$. The reason for this unaesthetic convention is that we do not wish projectors to harbor massless poles. To elucidate some of the potential subtleties, consider the derivation of the expression for $\Pi^{T,T\perp}$ in (2.13), starting with the definitions (2.2), (2.10), (2.11). By assumption, the states $|b_1^{(n),c}(k)\rangle$ form a complete (properly normalized) basis for one-particle states that can be created by $O^{T\perp,a}_{\mu\nu}$ from the vacuum. Therefore via insertion of the identity operator,

$$
\langle 0|O^{T\perp,a}_{\mu\nu}(x)O^{T\perp,b}_{\rho\sigma}(0)|0\rangle = \delta^{ab}\sum_{n,\varepsilon}\int\frac{d^3p}{(2\pi)^3}\frac{e^{-ip\cdot x}}{2E_n(\mathbf{p})}(f_b^{(n)})^2\ \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\bar{\varepsilon}^{(n)\alpha}_{(b)}p^{\beta}\ \epsilon_{\rho\sigma\delta\gamma}\varepsilon^{(n)\delta}_{(b)}p^{\gamma} \ , \tag{A.3}
$$

where $E_n(\mathbf{p}) = \sqrt{\mathbf{p}^2 + (m_b^{(n)})^2}$. The sum over (on-shell) polarizations yields

$$
\begin{aligned}
\frac{1}{2}\sum_{\varepsilon}\epsilon_{\mu\nu\alpha\beta}\bar{\varepsilon}^{(n)\alpha}_{(b)}p^{\beta}\ \epsilon_{\rho\sigma\delta\gamma}\varepsilon^{(n)\delta}_{(b)}p^{\gamma} &= (m^{(n)}_{(b)})^2\eta_{\mu[\rho}\eta_{\sigma]\nu} - (\eta_{\mu[\rho}p_{\sigma]}p_{\nu} - \eta_{\nu[\rho}p_{\sigma]}p_{\mu}) \\
&\equiv (\mathcal{P}^{\perp,(n)})_{\mu\nu,\rho\sigma} \ .
\end{aligned} \tag{A.4}
$$

Using Cauchy's theorem, we can convert (A.3) to an integral over four-momentum,

$$
\begin{aligned}
\langle 0|O^{T\perp,a}_{\mu\nu}(x)O^{T\perp,b}_{\rho\sigma}(0)|0\rangle &= \delta^{ab}\sum_{n}\int\frac{d^4p}{(2\pi)^4}\frac{ie^{-ip\cdot x}(f^{(n)}_{(b)})^2}{p^2 - (m^{(n)}_{(b)})^2}(\mathcal{P}^{\perp,(n)})_{\mu\nu,\rho\sigma} \\
&= i\delta^{ab}\int\frac{d^4p}{(2\pi)^4}e^{-ip\cdot x}(\mathcal{P}^{\perp})_{\mu\nu,\rho\sigma}\sum_{n}\frac{(f^{(n)}_{(b)})^2}{p^2 - (m^{(n)}_{(b)})^2} \ .
\end{aligned} \tag{A.5}
$$

Plugging this into (2.10) and using (2.11), we straightforwardly recover the expression for $\Pi^{T,T\perp}$ in (2.13).

In the second step of (A.5) we used the fact that $\oint dz f(a)/(z-a) = \oint dz f(z)/(z-a)$, which holds provided f is a holomorphic function in the region bounded by the contour. This allows us to take the projector for each mode off shell, so that it is a common factor which may be pulled out in front of the summand. This step would have failed if $\mathcal{P}$ was a properly normalized projector, since then it would have a pole inside the contour of integration.

B General solution to the free equation of motion for b_{MN}

The free equations of motion (3.8) for the two-form are

$$z\epsilon_{\mu\nu}{}^{\rho\sigma}\left(-2ik_\rho b_{\sigma z} + \partial_z b_{\rho\sigma}\right) - 2i\mu b_{\mu\nu} = 0 \ , \tag{B.1}$$
$$z\epsilon_\mu{}^{\nu\rho\sigma} k_\nu b_{\rho\sigma} + 2\mu b_{\mu z} = 0 \ , \tag{B.2}$$

where we have used the Fourier-transformed fields

$$b_{MN}(k,z) = \int d^4x e^{ik\cdot x} b_{MN}(x,z) \ . \tag{B.3}$$

Equation (B.2) yields the constraint

$$b_{\mu z} = -\frac{z}{2\mu}\epsilon_\mu{}^{\nu\rho\sigma} k_\nu b_{\rho\sigma} \ , \tag{B.4}$$

which, when plugged into the first equation of motion gives

$$z^2\left(k_\mu k^\rho b_{\rho\nu} - k_\nu k^\rho b_{\rho\mu} - k^2 b_{\mu\nu}\right) + \frac{i\mu z}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\partial_z b_{\rho\sigma} + \mu^2 b_{\mu\nu} = 0 \ , \tag{B.5}$$

or in terms of projectors,

$$\left[-z^2\mathcal{P}^\perp + \mu z(P^+ - P^-)\partial_z + \mu^2\right] b_{\mu\nu} = 0 \ . \tag{B.6}$$

We can project (B.6) onto its self-dual and anti-self-dual parts. For the projections of the $\mathcal{P}^\perp b$ term, we write $b = (P^+ + P^-)b$ and use (A.2) as necessary. The resulting two equations may be rearranged to the form

$$b^\mp = \frac{2}{k^4 z^2}\left(\pm\mu z\partial_z + \mu^2 - \frac{1}{2}k^2 z^2\right)(\mathcal{P}^\perp - \mathcal{P}^\parallel)b^\pm \ . \tag{B.7}$$

We can thus plug the equation for b^- into the equation for b^+, deriving an equation for b^+ alone, or vice versa. After some simplification and using $(\mathcal{P}^\perp - \mathcal{P}^\parallel)^2 = k^4\mathbf{1}$, we arrive at the equations derived in [18] and quoted in (3.10):

$$\left[z^2\partial_z^2 - z\partial_z + k^2 z^2 - \mu(\mu \pm 2)\right] b^\pm_{\mu\nu} = 0 \ . \tag{B.8}$$

The general solutions are

$$\begin{aligned} b^+_{\mu\nu}(k,z) &= \tilde{S}_{\mu\nu}(k) z J_{-\mu-1}(kz) + \tilde{s}_{\mu\nu}(k) z J_{\mu+1}(kz) \ , \\ b^-_{\mu\nu}(k,z) &= \tilde{A}_{\mu\nu}(k) z J_{-\mu+1}(kz) + \tilde{a}_{\mu\nu}(k) z J_{\mu-1}(kz) \ , \end{aligned} \tag{B.9}$$

where $(\tilde{S},\tilde{s})$ and $(\tilde{A},\tilde{a})$ are self-dual and anti-self-dual polarizations respectively, and J is a standard Bessel function. These solutions are appropriate (and convenient) for non-integer μ. For integer μ, the $J_{-\mu\mp1}$ terms should be replaced by $Y_{\mu\pm1}$. We must remember, however, that we started with first order equations (B.7). These equations will be satisfied on the solutions (B.9) if and only if

$$(\tilde{A},\tilde{a}) = \frac{1}{k^2}(\mathcal{P}^\perp - \mathcal{P}^\parallel)(\tilde{S},\tilde{s}) \quad \Rightarrow \quad (\tilde{S},\tilde{s}) = \frac{1}{k^2}(\mathcal{P}^\perp - \mathcal{P}^\parallel)(\tilde{A},\tilde{a}) \ . \tag{B.10}$$

Let us briefly comment on the bulk to boundary propagator for b. The form it takes depends on the sign of μ. If $\mu > 0$, then the $\tilde{S}$ term of (B.9) dominates as $z = \varepsilon \to 0$. The UV boundary condition matches b^+ onto a self-dual source, while the IR boundary condition sets b^- to zero:

$$b^+_{\mu\nu}(k,\varepsilon) = \frac{\ell^{\mu-1/2}}{\varepsilon^\mu} S_{\mu\nu}(k) \ , \qquad b^-_{\mu\nu}(k,z_0) = 0 \ , \qquad (\mu > 0) \ . \tag{B.11}$$

These conditions lead to the particular solution (3.25). On the other hand, if $\mu < 0$, the $\tilde{a}$ term dominates. In this case b^- should match onto an anti-self-dual source at the UV boundary and b^+ should be zero at the IR boundary:

$$b^-_{\mu\nu}(k,\varepsilon) = \frac{\ell^{|\mu|-1/2}}{\varepsilon^{|\mu|}} a_{\mu\nu}(k) \ , \qquad b^+_{\mu\nu}(k,z_0) = 0 \ , \qquad (\mu < 0) \ . \tag{B.12}$$

These boundary conditions lead to the particular solution

$$\begin{aligned} b^-_{\mu\nu}(k,z) &= \ell^{|\mu|-1/2} a_{\mu\nu}(k) \frac{B^-_<(k,z)}{\varepsilon^{|\mu|} B^-_<(k,\varepsilon)} \ , \\ b^+_{\mu\nu}(k,z) &= \ell^{|\mu|-1/2} \left[a_{\mu\nu} - \frac{2}{k^2} \left(k_\mu k^\rho a_{\rho\nu} - k_\nu k^\rho a_{\rho\mu} \right) \right] \frac{B^+_<(k,z)}{\varepsilon^{|\mu|} B^-_<(k,\varepsilon)} \ , \end{aligned} \tag{B.13}$$

where

$$\begin{aligned} B^\pm_<(k,z) &= z J_{-|\mu|\pm 1}(kz) - c^<_b(k,z_0) z J_{|\mu|\mp 1}(kz) \ , \qquad \text{with} \\ c^<_b(k,z_0) &= \frac{J_{-|\mu|+1}(kz_0)}{J_{|\mu|-1}(kz_0)} \ . \end{aligned} \tag{B.14}$$

These expressions are appropriate for the non-integer μ case. For integer μ, the $J_{-|\mu|\pm 1}$ should be replaced by $Y_{|\mu|\mp 1}$.

C Dual sources, correlation functions, and matrix elements

As usual in AdS/CFT, we determine the generating functional for the correlators of field theory operators by evaluating the supergravity action on the bulk-to-boundary propagators of the dual fields. The UV boundary conditions on these propagators are determined in such a way that for a 4d source $\phi_0(x)$, the dual five-dimensional field has boundary condition[10] $\phi(x, z = \varepsilon) = \phi_0(x)$. This determines the generating functional to be

$$Z[\phi_0(x)] = e^{i S_{sugra}(\phi)|_{\phi(\varepsilon)=\phi_0}} \ . \tag{C.1}$$

(Strictly speaking, this would be the leading saddle point approximation in a full quantum gravity dual description). We will usually work with momentum-space correlators. Functional differentiation in terms of momentum-space sources obeys

$$\frac{\delta}{\delta J(-k)} J(p) = (2\pi)^4 \delta^{(4)}(p-k) \ . \tag{C.2}$$

[10] Up to powers of ε, as discussed after equation (3.25).

We are mostly interested in correlators of the tensor operator $\bar{q}\sigma_{\mu\nu}q$, but we often work with the pseudo-tensor or (anti) self-dual versions discussed in Section 2. Here we review the definitions of the sources for these operators and the relations between them.

In the case $\mu > 0$ the bulk to boundary propagator is determined by a self-dual source, $P^+S = S$, which naturally couples to a self-dual operator in the field theory Lagrangian (in order to get a Lorentz scalar). The conjugate $\bar{S}_{\mu\nu}$ is anti-self-dual and naturally couples to the anti-self-dual, conjugate operator. These operators can be taken as

$$O^{\pm}_{\mu\nu}(x) = \bar{q}(x)\sigma_{\mu\nu}\frac{(1\pm\gamma_5)}{2}q(x) \ , \tag{C.3}$$

and one indeed has $(O^{T,+})^* = O^{T,-}$. From them one can construct tensor and pseudo-tensor operators,

$$O^{T}_{\mu\nu} = O^{+}_{\mu\nu} + O^{-}_{\mu\nu} = \bar{q}(x)\sigma_{\mu\nu}q(x) \ , \qquad O^{PT}_{\mu\nu} = O^{+}_{\mu\nu} - O^{-}_{\mu\nu} = \bar{q}(x)\sigma_{\mu\nu}\gamma_5 q(x) \ . \tag{C.4}$$

These should couple to a tensor source and pseudo-tensor source, $\mathcal{T}_{\mu\nu}$ and $\mathcal{P}_{\mu\nu}$, given by

$$\begin{array}{l} \mathcal{T}_{\mu\nu}(k) = \frac{1}{2}\left(S_{\mu\nu}(k) + \bar{S}_{\mu\nu}(-k)\right) \\ \mathcal{P}_{\mu\nu}(k) = \frac{1}{2}\left(S_{\mu\nu}(k) - \bar{S}_{\mu\nu}(-k)\right) \end{array} \quad \Rightarrow \quad \begin{array}{l} S_{\mu\nu}(k) = \mathcal{T}_{\mu\nu}(k) + \mathcal{P}_{\mu\nu}(k) \\ \bar{S}_{\mu\nu}(k) = \mathcal{T}_{\mu\nu}(-k) - \mathcal{P}_{\mu\nu}(-k) \end{array} \ , \quad (\mu > 0) \ . \tag{C.5}$$

Note that $\mathcal{P}, \mathcal{T}$ satisfy $\mathcal{P}_{\mu\nu} = \frac{i}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\mathcal{T}_{\rho\sigma}$, as do the corresponding operators, O^{PT} and O^{T}. The relative normalization between $\mathcal{T}, \mathcal{P}$ and $S, \bar{S}$ is fixed by the requirement that

$$S^{\mu\nu}O^{+}_{\mu\nu} + \bar{S}^{\mu\nu}O^{-}_{\mu\nu} = \mathcal{T}^{\mu\nu}O^{T}_{\mu\nu} + \mathcal{P}^{\mu\nu}O^{PT}_{\mu\nu} \ . \tag{C.6}$$

When $\mu < 0$ on the other hand, the bulk to boundary propagator is given in terms of an anti-self-dual source $a_{\mu\nu}$. This source naturally couples to $O^{T,-}$, while $\bar{a}_{\mu\nu}$ couples to $O^{T,+}$. The relationship of these sources to the tensor and pseudo-tensor sources introduced above is

$$\begin{array}{l} \mathcal{T}_{\mu\nu}(k) = \frac{1}{2}\left(a_{\mu\nu}(k) + \bar{a}_{\mu\nu}(-k)\right) \\ \mathcal{P}_{\mu\nu}(k) = \frac{1}{2}\left(\bar{a}_{\mu\nu}(k) - a_{\mu\nu}(-k)\right) \end{array} \quad \Rightarrow \quad \begin{array}{l} a_{\mu\nu}(k) = \mathcal{T}_{\mu\nu}(-k) - \mathcal{P}_{\mu\nu}(-k) \\ \bar{a}_{\mu\nu}(k) = \mathcal{T}_{\mu\nu}(k) + \mathcal{P}_{\mu\nu}(k) \end{array} \ , \quad (\mu < 0) \ . \tag{C.7}$$

(Note that (C.5) and (C.7) *do not* imply $a = \bar{S}$; these equations apply for different values of μ, corresponding to different dual Lagrangians).

We now derive the tensor-tensor two-point function from the free supergravity action. The same result was already derived in [18] , but we repeat it here for completeness.

We begin by determining the generating functional for the two-point functions. First assume that $\mu > 0$. Evaluating the action on the bulk-to-boundary propagator (3.25), the bulk and IR boundary contributions to the action vanish, and we are left with the UV boundary term,

$$S_{\rm sd} = \frac{1}{4g_b^2\ell}\int d^4x {\rm tr}\left[\bar{b}_{\mu\nu}b^{\mu\nu}\right]_{z=\varepsilon} = \frac{1}{4\ell g_b^2}\int d^4x {\rm tr}\left[\overline{b^+_{\mu\nu}}b^{-\mu\nu} + \overline{b^-_{\mu\nu}}b^{+\mu\nu}\right]_{z=\varepsilon} \ . \tag{C.8}$$

Plugging in (3.25), evaluated at $z=\varepsilon$ gives

$$\begin{aligned} S_{\rm sd} &= \frac{\ell^{2\mu-2}}{4g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_>^-(k,\varepsilon)}{\varepsilon^{2\mu}B_>^+(k,\varepsilon)}\text{tr}\left\{\bar{S}^{\alpha\beta}\left[S_{\alpha\beta}-\frac{2}{k^2}\left(k_\alpha k^\gamma S_{\gamma\beta}-k_\beta k^\gamma S_{\gamma\alpha}\right)\right]+c.c.\right\} \\ &= -\frac{2\ell^{2\mu-2}}{g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_>^-(k,\varepsilon)}{\varepsilon^{2\mu}B_>^+(k,\varepsilon)}\text{tr}\left[\frac{1}{k^2}k_\nu\bar{S}^{\nu\mu}k^\rho S_{\rho\mu}\right]\ , \end{aligned} \tag{C.9}$$

where we have used the fact that a self-dual tensor contracted with an ant-self-dual one vanishes. Since we are interested in the tensor-tensor two-point function, we choose to express the generating functional in terms of the tensor source. Using (C.5) with the constraint $\mathcal{P}_{\mu\nu}=\frac{i}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\mathcal{T}_{\rho\sigma}$, a short calculation shows that

$$k_\nu\bar{S}^{\nu\mu}k^\rho S_{\rho\mu} = -\frac{1}{2}\mathcal{T}^{\alpha\beta}(-k)\left[k^2\mathcal{T}_{\alpha\beta}(k)-2\left(k_\alpha k^\gamma\mathcal{T}_{\gamma\beta}(k)-k_\beta k^\gamma\mathcal{T}_{\gamma\alpha}(k)\right)\right]\ . \tag{C.10}$$

Plugging this into (C.9), we arrive at (3.27):

$$S_{\rm sd} = \frac{\ell^{2\mu-2}}{g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_>^-(k,\varepsilon)}{k^2\varepsilon^{2\mu}B_>^+(k,\varepsilon)}\text{tr}\left\{\mathcal{T}^{\alpha\beta}(-k)\left[\mathcal{P}^\perp_{\alpha\beta,\delta\gamma}-\mathcal{P}^\parallel_{\alpha\beta,\delta\gamma}\right]\mathcal{T}^{\delta\gamma}(k)\right\}\ . \tag{C.11}$$

Now suppose that $\mu<0$. In this case the on-shell action is

$$S_{\rm sd} = -\frac{1}{4\ell g_b^2}\int d^4x\,\text{tr}\left[\overline{b^+_{\mu\nu}}b^{-\mu\nu}+\overline{b^-_{\mu\nu}}b^{+\mu\nu}\right]_{z=\varepsilon}\ . \tag{C.12}$$

Plugging in the bulk to boundary propagator (B.13), we find

$$\begin{aligned} S_{\rm sd} &= -\frac{\ell^{2|\mu|-2}}{4g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_<^+(k,\varepsilon)}{\varepsilon^{2|\mu|}B_<^-(k,\varepsilon)}\text{tr}\left\{\bar{a}^{\alpha\beta}\left[a_{\alpha\beta}-\frac{2}{k^2}\left(k_\alpha k^\gamma a_{\gamma\beta}-k_\beta k^\gamma a_{\gamma\alpha}\right)\right]+c.c.\right\} \\ &= \frac{2\ell^{2|\mu|-2}}{g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_<^+(k,\varepsilon)}{\varepsilon^{2|\mu|}B_<^-(k,\varepsilon)}\text{tr}\left[\frac{1}{k^2}k_\nu\bar{a}^{\nu\mu}k^\rho a_{\rho\mu}\right]\ , \end{aligned} \tag{C.13}$$

Using (C.7) with $\mathcal{P}_{\mu\nu}=\frac{i}{2}\epsilon_{\mu\nu}{}^{\rho\sigma}\mathcal{T}_{\rho\sigma}$,

$$k_\nu\bar{a}^{\nu\mu}k^\rho a_{\rho\mu} = -\frac{1}{2}\mathcal{T}^{\alpha\beta}(k)\left[k^2\mathcal{T}_{\alpha\beta}(-k)-2\left(k_\alpha k^\gamma\mathcal{T}_{\gamma\beta}(-k)-k_\beta k^\gamma\mathcal{T}_{\gamma\alpha}(-k)\right)\right]\ , \tag{C.14}$$

and thus,

$$S_{\rm sd} = -\frac{\ell^{2|\mu|-2}}{g_b^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_<^+(k,\varepsilon)}{k^2\varepsilon^{2|\mu|}B_<^-(k,\varepsilon)}\text{tr}\left\{\mathcal{T}^{\alpha\beta}(-k)\left[\mathcal{P}^\perp_{\alpha\beta,\delta\gamma}-\mathcal{P}^\parallel_{\alpha\beta,\delta\gamma}\right]\mathcal{T}^{\delta\gamma}(k)\right\}\ . \tag{C.15}$$

The two cases $\mu>0$, $\mu<0$ can be combined as follows. Define the functions

$$\begin{aligned} B_n(k,z) &= \begin{cases} zJ_{-|\mu|+1}(kz)-c_b(k,z_0)zJ_{|\mu|-1}(kz)\ , & \mu\text{ non-integer,} \\ zY_{|\mu|-1}(kz)-c_b(k,z_0)zJ_{|\mu|-1}(kz)\ , & \mu\text{ integer,} \end{cases} \\ B_d(k,z) &= \begin{cases} zJ_{-|\mu|-1}(kz)-c_b(k,z_0)zJ_{|\mu|+1}(kz)\ , & \mu\text{ non-integer,} \\ zY_{|\mu|+1}(kz)-c_b(k,z_0)zJ_{|\mu|+1}(kz)\ , & \mu\text{ integer,} \end{cases} \end{aligned} \tag{C.16}$$

with

$$c_b(k,z_0) = \begin{cases} \frac{J_{-|\mu|+1}(kz_0)}{J_{|\mu|-1}(kz_0)} \ , & \mu \text{ non-integer,} \\ \\ \frac{Y_{|\mu|-1}(kz_0)}{J_{|\mu|-1}(kz_0)} \ , & \mu \text{ integer,} \end{cases} \tag{C.17}$$

for both positive and negative μ. Observe that when $\mu > 0$, $B_n = B_>^-$, and when $\mu < 0$, $B_n = B_<^+$. Similarly, when $\mu > 0$, $B_d = B_>^+$ and when $\mu < 0$, $B_d = B_<^-$. Thus, (C.11) and (C.15) may be summarized as

$$S_{\text{sd}} = \text{sgn}(\mu)\frac{\ell^{2|\mu|-2}}{g_b^2}\int \frac{d^4k}{(2\pi)^4}\frac{B_n(k,\varepsilon)}{k^2\varepsilon^{2|\mu|}B_d(k,\varepsilon)}\text{tr}\left\{\mathcal{T}^{\alpha\beta}(-k)\left[\mathcal{P}^{\perp}_{\alpha\beta,\delta\gamma} - \mathcal{P}^{\parallel}_{\alpha\beta,\delta\gamma}\right]\mathcal{T}^{\delta\gamma}(k)\right\}, \tag{C.18}$$

valid for all μ. This is the expression we can functionally differentiate to compute the O^T-O^T two-point correlator. Due to the relation between $\mathcal{P}, \mathcal{T}$ discussed above, the $\langle O^{PT} O^{PT}\rangle$ and $\langle O^T O^{PT}\rangle$ correlators follow from the tensor-tensor correlator.

The two-point correlator is computed via the usual prescription:

$$\langle O^T_{\mu\nu}(p) O^T_{\rho\sigma}(-q)\rangle = \frac{\delta}{\delta\mathcal{T}^{\mu\nu}(-p)}\frac{\delta}{\delta\mathcal{T}^{\rho\sigma}(q)} iS_{\text{sd}} \ . \tag{C.19}$$

Evaluating the right-hand side, restoring isospin indices, leads to

$$\langle O^{T,a}_{\mu\nu}(p) O^{T,b}_{\rho\sigma}(-q)\rangle = i(2\pi)^4\delta^{(4)}(p-q)\delta^{ab}\text{sgn}(\mu)\frac{4\ell^{2|\mu|-2}B_n(q,\varepsilon)}{g_b^2 q^2\varepsilon^{2|\mu|}B_d(q,\varepsilon)}\left(\mathcal{P}^{\perp}_{\mu\nu,\rho\sigma} - \mathcal{P}^{\parallel}_{\mu\nu,\rho\sigma}\right) \ , \tag{C.20}$$

which gives a matrix element

$$\Pi^{T,T,ab}_{\mu\nu,\rho\sigma}(k) = \delta^{ab}\text{sgn}(\mu)\frac{4\ell^{2|\mu|-2}B_n(k,\varepsilon)}{g_b^2 k^2\varepsilon^{2|\mu|}B_d(k,\varepsilon)}\left[\mathcal{P}^{\perp}_{\mu\nu,\rho\sigma} - \mathcal{P}^{\parallel}_{\mu\nu,\rho\sigma}\right] \ . \tag{C.21}$$

Given (2.11), we have $\Pi^{T,T\parallel} = -\Pi^{T,T\perp}$, with

$$\Pi^{T,T\perp}(k) = \text{sgn}(\mu)\frac{4\ell^{2|\mu|-2}B_n(k,\varepsilon)}{g_b^2 k^2\varepsilon^{2|\mu|}B_d(k,\varepsilon)} \ . \tag{C.22}$$

Now, finally, we take the $\varepsilon \to 0$ limit. We will first consider the case of non-integer μ, so that

$$\frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|}B_d(k,\varepsilon)} = \frac{J_{-|\mu|+1}(k\varepsilon) - c_b(k,z_0)J_{|\mu|-1}(k\varepsilon)}{\varepsilon^{2|\mu|}\left[J_{-|\mu|-1}(k\varepsilon) - c_b(k,z_0)J_{|\mu|+1}(k\varepsilon)\right]} \ , \tag{C.23}$$

For convenience, define $B_{n,d}(k,z) = z\tilde{B}_{n,d}(k,z)$. We have that $B_n/B_d = \tilde{B}_n/\tilde{B}_d$. Using series expansions of the Bessel functions, we find that the leading ε behavior in the denominator is

$$\varepsilon^{2|\mu|}\tilde{B}_d(k,\varepsilon) = \frac{2^{|\mu|+1}}{\Gamma(-|\mu|)}k^{-|\mu|-1}\varepsilon^{|\mu|-1}\left(1 + \mathcal{O}(\varepsilon^2)\right) \ . \tag{C.24}$$

Meanwhile in the numerator we have

$$\tilde{B}_n(k,\varepsilon) = \frac{2^{|\mu|-1}}{\Gamma(-|\mu|+2)} k^{-|\mu|+1} \varepsilon^{-|\mu|+1} \left(1 + \mathcal{O}(\varepsilon^2)\right) + \\ - \frac{c_b(k,z_0)}{2^{|\mu|-1}\Gamma(|\mu|)} k^{|\mu|-1} \varepsilon^{|\mu|-1} \left(1 + \mathcal{O}(\varepsilon^2)\right) \ . \tag{C.25}$$

Hence the ratio takes the form

$$\frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|} B_d(k,\varepsilon)} = \frac{\Gamma(-|\mu|)}{4\Gamma(-|\mu|+2)} k^2 \varepsilon^{-2(|\mu|+1)} \left(1 + \mathcal{O}(\varepsilon^2)\right) + \\ - c_b(k,z_0) \frac{\Gamma(-|\mu|)}{2^{2|\mu|}\Gamma(|\mu|)} k^{2|\mu|} \left(1 + \mathcal{O}(\varepsilon^2)\right) \ . \tag{C.26}$$

Observe that all of the terms in the first series go as ε to a negative or positive power; there are no terms that go as ε^0 for non-integer μ. The terms that diverge with ε represent contact terms which we are not interested in. Hence, the first series can be ignored in the $\varepsilon \to 0$ limit. Meanwhile, only the first term of the second series survives the limit. Hence,

$$\lim_{\varepsilon \to 0} \frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|} B_d(k,\varepsilon)} = \text{ contact terms } - \frac{\Gamma(-|\mu|)}{2^{2|\mu|}\Gamma(|\mu|)} k^{2|\mu|} c_b(k,z_0) \ . \tag{C.27}$$

Next consider the integer μ case. We have

$$\frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|} B_d(k,\varepsilon)} = \frac{Y_{|\mu|-1}(k\varepsilon) - c_b(k,z_0) J_{|\mu|-1}(k\varepsilon)}{\varepsilon^{2|\mu|} \left[Y_{|\mu|+1}(k\varepsilon) - c_b(k,z_0) J_{|\mu|+1}(k\varepsilon)\right]} \tag{C.28}$$

For integer n, the Bessel function Y_n is defined by the series

$$Y_n(x) = -\frac{1}{\pi}\left(\frac{x}{2}\right)^{-n} \sum_{j=0}^{n-1} \frac{(n-j-1)!}{j!} \left(\frac{x}{2}\right)^{2j} + \frac{2}{\pi} \log\left(\frac{x}{2}\right) J_n(x) + \\ - \frac{1}{\pi}\left(\frac{x}{2}\right)^{n} \sum_{j=0}^{\infty} \frac{[\psi_0(j+1) + \psi_0(n+j+1)]}{j!(n+j)!} \left(\frac{-x}{2}\right)^{2j} \ , \tag{C.29}$$

where ψ_0 is the digamma function. For integer values, $\psi_0(n) = -\gamma + H_{n-1} = -\gamma + \sum_{j=1}^{n-1} \frac{1}{j}$, where γ is the Euler-Mascheroni constant, and H_n a harmonic number. We note that the first series begins at order x^{-n} and terminates at order x^{n-1}. The J_n term and the second series begin at order x^n. In the denominator then, the leading divergence is

$$\varepsilon^{2|\mu|} \tilde{B}_d(k,\varepsilon) = -\frac{2^{|\mu|+1}|\mu|!}{\pi} k^{-|\mu|-1} \varepsilon^{|\mu|-1} \left(1 + \mathcal{O}(\varepsilon^2)\right) \ . \tag{C.30}$$

In the numerator, all terms in the first series of $Y_{|\mu|-1}$ range from $\varepsilon^{-|\mu|+1}$ to $\varepsilon^{|\mu|-2}$, and hence represent contact terms. The terms we are interested in, given (C.30), go as $\varepsilon^{|\mu|-1}$. They are the leading terms in $J_{|\mu|-1}$, (both the explicit $J_{|\mu|-1}$ in (C.28) and the one contained in (C.29)), as well as the first term in the series on the second line of (C.29):

$$\tilde{B}_n(k,\varepsilon) = \cdots + \left[\frac{2}{\pi}\log\left(\frac{k\varepsilon}{2}\right) - c_b(k,z_0)\right] \frac{1}{2^{|\mu|-1}(|\mu|-1)!} k^{|\mu|-1} \varepsilon^{|\mu|-1} \left(1 + \mathcal{O}(\varepsilon^2)\right) + \\ - \frac{(H_{|\mu|-1} - 2\gamma)}{\pi 2^{|\mu|-1}(|\mu|-1)!} k^{|\mu|-1} \varepsilon^{|\mu|-1} \left(1 + \mathcal{O}(\varepsilon^2)\right) \ , \tag{C.31}$$

where the $\cdots$ represent the contact terms. Thus we have that

$$\lim_{\varepsilon\to 0}\frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|}B_d(k,\varepsilon)} = \text{contact terms} + \\ -\frac{\pi}{2^{2|\mu|}|\mu|!(|\mu|-1)!}\left[\frac{2}{\pi}\log\left(\frac{k\varepsilon}{2}\right) - c_b(k,z_0) - \frac{1}{\pi}(H_{|\mu|-1}-2\gamma)\right]k^{2|\mu|}\ . \tag{C.32}$$

In a standard renormalization scheme with renormalization scale $M_r = \ell^{-1}$, this becomes

$$\lim_{\varepsilon\to 0}\frac{B_n(k,\varepsilon)}{\varepsilon^{2|\mu|}B_d(k,\varepsilon)} = \text{contact terms} + \frac{1}{2^{2|\mu|}|\mu|!(|\mu|-1)!}\left[\pi c_b(k,z_0) - \log\left(k^2\ell^2\right)\right]k^{2|\mu|}\ . \tag{C.33}$$

In summary, (dropping the contact terms), we have

$$\Pi^{T,T\perp}(k) = \begin{cases} -\frac{\mathrm{sgn}(\mu)\Gamma(-|\mu|)}{2^{2|\mu|-2}g_b^2\Gamma(|\mu|)}c_b(k,z_0)(k\ell)^{2|\mu|-2}\ , & \mu \text{ non-integer,} \\ \frac{\mathrm{sgn}(\mu)}{2^{2|\mu|-2}g_b^2|\mu|!(|\mu|-1)!}\left[\pi c_b(k,z_0) - \log\left(k^2\ell^2\right)\right](k\ell)^{2|\mu|-2}\ , & \mu \text{ integer.} \end{cases} \tag{C.34}$$

This reproduces (3.28) for $\mu > 0$. Plugging in (C.17), we recover (3.31).

D Spectrum of normalizable modes

In this case we are interested in the normalizable part of the solution, (B.9). In the following we will consider the $\mu > 0$ case only; an analogous treatment can be performed for the $\mu < 0$ case. For $\mu > 0$, we restrict to the normalizable component of (B.9) by setting $S = A = 0$, resulting in

$$b^+_{\mu\nu}(k,z) = \tilde{s}_{\mu\nu}(k)zJ_{\mu+1}(mz)\ , \qquad b^-_{\mu\nu}(k,z) = \left[\tilde{s}_{\mu\nu} + \frac{4}{k^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho\right]zJ_{\mu-1}(mz)\ . \tag{D.1}$$

Since $b_{\mu\nu} = b^+_{\mu\nu} + b^-_{\mu\nu}$ and $b_{\mu z} = -\frac{z}{2\mu}\epsilon_\mu{}^{\nu\rho\sigma}k_\nu b_{\rho\sigma}$, this implies

$$b_{\mu\nu}(k,z) = \frac{2\mu}{m}\tilde{s}_{\mu\nu}J_\mu(mz) + \frac{4}{k^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho zJ_{\mu-1}(mz)\ , \tag{D.2}$$

$$b_{\mu z}(k,z) = -\frac{1}{m}\epsilon_\mu{}^{\nu\rho\sigma}k_\nu\tilde{s}_{\rho\sigma}zJ_\mu(mz) = \frac{2i}{m}\tilde{s}_{\mu\nu}k^\nu zJ_\mu(mz)\ . \tag{D.3}$$

These solutions solve (B.6)-(B.8) with eigenvalue $k^2 \to m^2$. The IR Neumann-like boundary condition, $b^-(z_0) = 0$, fixes the eigenvalues to be $m_n = x_{\mu-1,n}/z_0$. For each eigenvalue, there is a corresponding self-dual polarization $\tilde{s}^{(n)}_{\mu\nu}(k)$, which contains six real degrees of freedom.

We would like to evaluate (the quadratic part of) (3.7) on a sum over the eigenmodes. The boundary terms vanish since we have restricted to normalizable modes and imposed the IR boundary condition. To evaluate the bulk part of the action, we require the components

of db and $\star b$, evaluated on the solution (D.2). Working in momentum space we have

$$\begin{aligned}
(db)_{\mu\nu\rho} &= 3\partial_{[\mu}b_{\nu\rho]} \to -i(k_\mu b_{\nu\rho} + k_\nu b_{\rho\mu} + k_\rho b_{\mu\nu}) \\
&= -\frac{2i\mu}{m}(k_\mu \tilde{s}_{\nu\rho} + k_\nu \tilde{s}_{\rho\mu} + k_\rho \tilde{s}_{\mu\nu})J_\mu(mz) \ , \qquad (D.4) \\
(db)_{\mu\nu z} \to & -i(k_\mu b_{\nu z} - k_\nu b_{\mu z}) + \partial_z b_{\mu\nu} \\
&= \frac{4}{m}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho z J_\mu(mz) + \frac{2\mu}{m}\tilde{s}_{\mu\nu}\partial_z J_\mu(mz) + \frac{4}{k^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho \partial_z[zJ_{\mu-1}(mz)] \\
&= \frac{2\mu}{m}\tilde{s}_{\mu\nu}\partial_z J_\mu(mz) + 4k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho \left[\frac{z}{m}J_\mu(mz) + \frac{1}{k^2}\partial_z[zJ_{\mu-1}(mz)]\right] \ , \qquad (D.5)
\end{aligned}$$

while the components of $\frac{i\mu}{\ell}\star b$ are

$$\begin{aligned}
\frac{i\mu}{\ell}(\star b)_{\mu\nu\rho} &= \frac{i\mu}{2z}\cdot 2\epsilon_{\mu\nu\rho}{}^\sigma b_{\sigma z} \to -\frac{2\mu}{m}\epsilon_{\mu\nu\rho}{}^\sigma \tilde{s}_{\sigma\alpha}k^\alpha J_\mu(mz) \\
&= -\frac{i\mu}{m}k_\alpha \epsilon_{\mu\nu\rho\sigma}\epsilon^{\sigma\alpha\beta\gamma}\tilde{s}_{\beta\gamma}J_\mu(mz) = -\frac{2i\mu}{m}(k_\mu \tilde{s}_{\nu\rho} + k_\nu \tilde{s}_{\rho\mu} + k_\rho \tilde{s}_{\mu\nu})J_\mu(mz) \ , \qquad (D.6) \\
\frac{i\mu}{\ell}(\star b)_{\mu\nu z} &= -\frac{i\mu}{2z}\epsilon_{\mu\nu}{}^{\rho\sigma}b_{\rho\sigma} \to -\frac{2\mu^2}{mz}\tilde{s}_{\mu\nu}J_\mu(mz) - \frac{2i\mu}{k^2}\epsilon_{\mu\nu}{}^{\rho\sigma}k_\rho \tilde{s}_{\sigma\alpha}k^\alpha J_{\mu-1}(mz) \\
&= -\frac{2\mu^2}{mz}\tilde{s}_{\mu\nu}J_\mu(mz) + \frac{\mu}{k^2}k^\rho k_\alpha \epsilon_{\mu\nu\rho\sigma}\epsilon^{\sigma\alpha\beta\gamma}\tilde{s}_{\beta\gamma}J_{\mu-1}(mz) \\
&= \left[-\frac{2\mu^2}{mz}J_\mu(mz) + 2\mu J_{\mu-1}(mz)\right]\tilde{s}_{\mu\nu} + \frac{4\mu}{k^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho J_{\mu-1}(mz) \ . \qquad (D.7)
\end{aligned}$$

Therefore we see that

$$\left(db - \frac{i\mu}{\ell}\star b\right)_{\mu\nu\rho} = 0 \ , \qquad (D.8)$$

while

$$\begin{aligned}
\left(db - \frac{i\mu}{\ell}\star b\right)_{\mu\nu z} &= 2\mu\left[\frac{1}{m}\partial_z J_\mu(mz) + \frac{\mu}{mz}J_\mu(mz) - J_{\mu-1}(mz)\right]\tilde{s}_{\mu\nu} + \\
&\quad + \left[\frac{k^2 z}{m}J_\mu(mz) + \partial_z[zJ_{\mu-1}(mz)] - \mu J_{\mu-1}(mz)\right]\frac{4}{k^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho \\
&= \frac{4(k^2-m^2)}{mk^2}k_{[\mu}\tilde{s}_{\nu]\rho}k^\rho z J_\mu(mz) \ , \qquad (D.9)
\end{aligned}$$

after making use of Bessel function identities.

Then we have

$$\begin{aligned}
\bar{b}\wedge\left(db - \frac{i\mu}{\ell}\star b\right) &= \frac{1}{2!3!}\bar{b}_{MN}\left(db - \frac{i\mu}{\ell}\star b\right)_{PQR} e^{MNPQR}d^5x \\
&= \frac{1}{4}\bar{b}_{\mu\nu}\left(db - \frac{i\mu}{\ell}\star b\right)_{\rho\sigma z}\epsilon^{\mu\nu\rho\sigma}d^5x \\
&= \frac{1}{4}\cdot\frac{2\mu}{m'}\,\overline{\tilde{s}'}_{\mu\nu}(k)J_\mu(m'z)\left[\frac{4(k^2-m^2)}{mk^2}k_{[\rho}\tilde{s}_{\sigma]\alpha}k^\alpha z J_\mu(mz)\right]\epsilon^{\mu\nu\rho\sigma}d^5x \\
&= -\frac{4i\mu}{mm'}\left(k^\mu \overline{\tilde{s}'}_{\mu\rho}\right)\frac{(k^2-m^2)}{k^2}\left(k_\nu \tilde{s}^{\nu\rho}\right) zJ_\mu(m'z)J_\mu(mz)d^5x \ . \qquad (D.10)
\end{aligned}$$

In the third step we plugged in some other, primed eigenmode for $\bar{b}$ and noted that the longitudinal part of $\bar{b}$ does not contribute to the contraction. We must multiply this result by $-i$ when plugging into the action, and therefore the conjugate term gives a symmetric contribution, interchanging primed and unprimed modes. The IR boundary condition leads to an orthogonal spectrum,

$$\int_0^{z_0} dz z J_\mu(m_n z) J_\mu(m_{n'} z) = \frac{z_0^2}{2} J_\mu(x_{\mu-1,n})^2 \delta_{nn'} \ , \tag{D.11}$$

and therefore, defining canonically normalized polarizations

$$s_{\mu\nu}^{(n)}(k) = \frac{z_0 J_\mu(x_{\mu-1,n})}{g_b m_n} \sqrt{\frac{2\mu}{\ell}} \ \tilde{s}_{\mu\nu}^{(n)}(k) \ , \tag{D.12}$$

the free action in the b-sector takes the form

$$S_{\rm sd} = -\sum_n \int \frac{d^4k}{(2\pi)^4} \mathrm{tr} \left\{ \left(k^\mu \bar{s}_{\mu\rho}^{(n)} \right) \frac{(k^2 - m_n^2)}{k^2} \left(k_\nu s^{(n)\nu\rho} \right) \right\} \ . \tag{D.13}$$

Note that, since $[b] = 3/2$, we have $[\tilde{s}] = 5/2$ and therefore $[s] = 1$.

It is natural that only the longitudinal components of s should enter into (D.13): s is a self-dual tensor, so the transverse components are not independent degrees of freedom. The six real degrees of freedom contained in s may be taken, in the rest frame, as $\mathrm{Re}(s_{0i})$ and $\mathrm{Im}(s_{0i})$. According to our charge and parity assignments for b, discussed in section 4, the imaginary-longitudinal (equivalently, real-transverse) components of s represent h_1/b_1 modes, while the real-longiudinal (equivalently, imaginary-transverse) components of s represent tensor ω/ρ modes, (2.3). Such modes can be given a canonical quadratic action by embedding them into s according to

$$s_{\mu\nu}^{(n)}(k) = P_{\mu\nu}^{+\alpha\beta} \left[-\frac{2i}{k} \epsilon_{\alpha\beta}{}^{\delta\gamma} k_\delta \left(b_{1\gamma}^{(n)}(k) + i\rho_\gamma^{T,(n)}(k) \right) \right] \ . \tag{D.14}$$

As we discussed, at the level of the free action, these modes have identical spectra.

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